You’re the Designer
Research studies are designed to gather and analyze data in order to answer questions. The results are displayed in tables and graphs.
**Vocabulary**

Match each term on the left with a definition on the right.

1. difference  A. the result of an addition
2. factor  B. a number that is multiplied by another number to get a product
3. natural numbers  C. numbers that can be expressed in the form \( \frac{a}{b} \), where \( a \) and \( b \) are both integers and \( b \neq 0 \)
4. ratio  D. the result of a subtraction
5. sum  E. a comparison of two quantities by division
F. the counting numbers: 1, 2, 3, ...

**Solve Proportions**

Solve each proportion.

6. \( \frac{3}{4} = \frac{x}{12} \)  7. \( \frac{15}{9} = \frac{3}{x} \)  8. \( \frac{10}{20} = \frac{x}{100} \)  9. \( \frac{250}{1500} = \frac{x}{100} \)

**Compare and Order Real Numbers**

Compare. Write <, >, or =.

10. 20 13  11. \( \frac{2}{3} \) \( \frac{1}{2} \)  12. \( \frac{3}{4} \) \( \frac{7}{9} \)  13. 0.75 \( \frac{9}{12} \)

Order the numbers from least to greatest.

14. \( \frac{1}{2} \) \( \frac{4}{5} \) \( \frac{1}{8} \) \( \frac{3}{4} \) \( \frac{2}{3} \)  15. 0.12, \( \frac{2}{5} \) \( \frac{3}{4} \), 0.3, \( \frac{1}{3} \)

**Multiply Decimals**

Multiply.

16. 0.25 \( \times \) 300  17. 0.5 \( \times \) 4000  18. 0.05 \( \times \) 200  19. 0.125 \( \times \) 9600

**Divide Decimals**

Divide.

20. 435 \( \div \) 10  21. 32 \( \div \) 100  22. 777 \( \div \) 1000  23. 295 \( \div \) 10,000

**Fractions, Decimals, and Percents**

Write the equivalent decimal.

24. \( \frac{3}{5} \)  25. 45%  26. \( \frac{3}{4} \)  27. 8%

Write the equivalent percent.

28. \( \frac{1}{4} \)  29. 0.2  30. 0.36  31. \( \frac{1}{10} \)
Bar and Circle Graphs

Data displayed in bar graphs and circle graphs can be used to solve equations. In these problems, parts of the graphs are missing.

Example 1

The top part of this graph was torn off. If Warren received 15% of the votes, how many votes did Adams receive?

Step 1 Find the total number of votes. Let \( t \) represent the total.

\[
15\% \text{ of the total votes is } 42 \text{ votes.}
\]

\[
0.15 \cdot t = 42
\]

\[
t = 280 \text{ votes}
\]

Step 2 Find the number of votes Adams received.

Let \( a \) represent the number of votes received by Adams.

Let \( h, w, s, \) and \( m \) represent the number of votes received by Hansen, Warren, Sweeney, and Marino.

\[
t = a + h + w + s + m
\]

\[
280 = a + 52 + 42 + 65 + 28
\]

\[
280 = a + 187
\]

\[
-187 \quad -187
\]

\[
93 = a
\]

Adams received 93 votes.

Try This

1. The missing bar is twice as tall as the bar for week 2. How many total miles did Kim bike in these five weeks?

2. People aged 20–29 years walked 275 more miles than the oldest age group. Find the total miles walked by all age groups.
Remember that a circle graph represents all the data in a data set. The percent represented by each section is a part of the whole data set, so the sum of all the percents must be 100%.

**Example 2**

A survey asked people in a neighborhood to agree or disagree with the following statement:

“We need a traffic light at Jefferson Avenue and Third Street.”

If 35% of the people disagreed with the statement, how many people had no opinion?

The number of people who answered “no opinion” is missing from the graph.

**Step 1** Find the total number of people who answered the survey. Let $t$ represent the total number of people.

\[
35\% \text{ of the total number of people } = 63 \text{ people.}
\]

\[
0.35 \cdot t = 63
\]

\[
t = 180 \text{ people}
\]

**Step 2** Find the number of people who answered “no opinion.”

Let $n$ represent the number of “no opinion” answers. Let $d$, $s$, $g$, and $a$ represent the number of “disagree,” “strongly disagree,” “strongly agree,” and “agree” answers.

\[
t = n + d + s + g + a
\]

\[
180 = n + 63 + 45 + 36 + 27
\]

\[
180 = n + 171
\]

\[
-171 -171
\]

\[
9 = n
\]

There were 9 people who had no opinion.

**Try This**

3. The students in a junior high school voted on their choice for a field trip. Sixteen students voted for the natural history museum. How many students voted for the winning choice?

4. At the fall dance recital, 40% of the tickets were sold to adults. What percent of the sales were to seniors?
Organizing and Displaying Data

**Objectives**
Organize data in tables and graphs.
Choose a table or graph to display data.

**Vocabulary**
bar graph
line graph
circle graph

**Who uses this?**
Nutritionists can display health information about food in bar graphs.

**Bar graphs, line graphs, and circle graphs**
can be used to present data in a visual way.

A **bar graph** displays data with vertical or horizontal bars. Bar graphs are a good way to display data that can be organized into categories. Using a bar graph, you can quickly compare the categories.

**Example 1**
Reading and Interpreting Bar Graphs

Use the graph to answer each question.

![Fat Content of a Sub Sandwich](image)

**A** Which ingredient contains the most fat?
*mayonnaise*

_The bar for mayonnaise is the longest._

**B** How many more grams of fat are in ham than in turkey?
6 − 3 = 3

_There are 6 grams of fat in ham and 3 grams of fat in turkey._

**C** How many total fat grams are in this sandwich?
1 + 6 + 3 + 9 + 11 = 30

_Add the number of fat grams for each ingredient._

**D** What percent of the total fat grams in this sandwich are from turkey?
\[
\frac{3}{30} = \frac{1}{10} = 10\%
\]

_Out of 30 total fat grams, 3 fat grams are from turkey._

Use the graph to answer each question.

1a. Which ingredient contains the least amount of fat?
1b. Which ingredients contain at least 8 grams of fat?
A double-bar graph can be used to compare two data sets. A double-bar graph has a key to distinguish between the two sets of data.

### Example 2

**Reading and Interpreting Double Bar Graphs**

Use the graph to answer each question.

**A** In which year did State College have the greatest average attendance for basketball?

- 2003

*Find the tallest orange bar.*

**B** On average, how many more people attended a football game than a basketball game in 2001?

- 20,000

- 13,000

- $7000 = 7000$

*Find the height of each bar for 2001 and subtract.*

### Check It Out!

2. Use the graph to determine which years had the same average basketball attendance. What was the average attendance for those years?

A **line graph** displays data using line segments. Line graphs are a good way to display data that changes over a period of time.

### Example 3

**Reading and Interpreting Line Graphs**

Use the graph to answer each question.

**A** At what time was the temperature the warmest?

- 4:00 P.M.

*Identify the highest point.*

**B** During which 4-hour time period did the temperature increase the most?

- From 8:00 A.M. to noon

*Look for the segment with the greatest positive slope.*

### Check It Out!

3. Use the graph to estimate the difference in temperature between 4:00 A.M. and noon.
A double-line graph can be used to compare how two related data sets change over time. A double-line graph has a key to distinguish between the two sets of data.

**Example 4**

**Reading and Interpreting Double-Line Graphs**

Use the graph to answer each question.

A double-line graph can be used to compare how two related data sets change over time. A double-line graph has a key to distinguish between the two sets of data.

**A** In which month(s) did airline B charge more than airline A?
April and September

**B** During which month(s) did the airlines charge the same airfare?
May

**Check it Out!**

4. Use the graph to describe the general trend of the data.

A **circle graph** shows parts of a whole. The entire circle represents 100% of the data and each sector represents a percent of the total. Circle graphs are good for comparing each category of data to the whole set.

**Example 5**

**Reading and Interpreting Circle Graphs**

Use the graph to answer each question.

**A** Which two fruits together make up half of the fruit salad?
bananas and strawberries

**B** Which fruit is used more than any other?
cantaloupe

**Check it Out!**

5. Use the graph to determine what percent of the fruit salad is cantaloupe.
Example 6

Choosing and Creating an Appropriate Display

Use the given data to make a graph. Explain why you chose that type of graph.

A Livestock Show Entries

<table>
<thead>
<tr>
<th>Animal</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicken</td>
<td>38</td>
</tr>
<tr>
<td>Goat</td>
<td>10</td>
</tr>
<tr>
<td>Horse</td>
<td>32</td>
</tr>
<tr>
<td>Pig</td>
<td>12</td>
</tr>
<tr>
<td>Sheep</td>
<td>25</td>
</tr>
</tbody>
</table>

A bar graph is appropriate for this data because it will be a good way to compare categories.

Step 1 Determine an appropriate scale and interval. The scale must include all of the data values. The scale is separated into equal parts, called intervals.

Step 2 Use the data to determine the lengths of the bars. Draw bars of equal width. The bars should not touch.

Step 3 Title the graph and label the horizontal and vertical scales.

B Division of Crops

<table>
<thead>
<tr>
<th>Crop</th>
<th>Area (acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>70</td>
</tr>
<tr>
<td>Fallow</td>
<td>50</td>
</tr>
<tr>
<td>Mixed vegetables</td>
<td>10</td>
</tr>
<tr>
<td>Soybeans</td>
<td>40</td>
</tr>
<tr>
<td>Wheat</td>
<td>30</td>
</tr>
</tbody>
</table>

A circle graph is appropriate for this data because it shows categories as parts of a whole.

Step 1 Calculate the percent of the total represented by each category.

Corn: \( \frac{70}{200} = 0.35 = 35\% \)
Soybeans: \( \frac{40}{200} = 0.2 = 20\% \)
Fallow: \( \frac{50}{200} = 0.25 = 25\% \)
Wheat: \( \frac{30}{200} = 0.15 = 15\% \)
Mixed vegetables: \( \frac{10}{200} = 0.05 = 5\% \)

Step 2 Find the angle measure for each sector of the graph. Since there are 360° in a circle, multiply each percent by 360°.

Corn: \( 0.35 \times 360° = 126° \)
Fallow: \( 0.25 \times 360° = 90° \)
Mixed vegetables: \( 0.05 \times 360° = 18° \)
Soybeans: \( 0.2 \times 360° = 72° \)
Wheat: \( 0.15 \times 360° = 54° \)

Step 3 Use a compass to draw a circle. Mark the center and use a straightedge to draw one radius. Then use a protractor to draw each central angle.

Step 4 Title the graph and label each sector.
Use the given data to make a graph. Explain why you chose that type of graph.

C  Chinnick College Enrollment

<table>
<thead>
<tr>
<th>Year</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1930</td>
<td>586</td>
</tr>
<tr>
<td>1955</td>
<td>2,361</td>
</tr>
<tr>
<td>1980</td>
<td>15,897</td>
</tr>
<tr>
<td>2005</td>
<td>21,650</td>
</tr>
</tbody>
</table>

A line graph is appropriate for this data because it will show the change in enrollment over a period of time.

Step 1  Determine the scale and interval for each set of data. Time should be plotted on the horizontal axis because it is independent.

Step 2  Plot a point for each pair of values. Connect the points using line segments.

Step 3  Title the graph and label the horizontal and vertical scales.

6. Use the given data to make a graph. Explain why you chose that type of graph.

The data below shows how Vera spends her time during a typical 5-day week during the school year.

Vera's Schedule

<table>
<thead>
<tr>
<th>Activity</th>
<th>Sleeping</th>
<th>Eating</th>
<th>School</th>
<th>Sports</th>
<th>Homework</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>45</td>
<td>8</td>
<td>30</td>
<td>10</td>
<td>10</td>
<td>17</td>
</tr>
</tbody>
</table>

THINK AND DISCUSS

1. What are some comparisons you can make by looking at a bar graph?
2. Name some key components of a good line graph.
3. GET ORGANIZED  Copy and complete the graphic organizer. In each box, tell which kind of graph is described.
GUIDED PRACTICE

Vocabulary  Use the vocabulary from this lesson to answer the following questions.

1. In a circle graph, what does each sector represent?
2. In a line graph, how does the slope of a line segment relate to the rate of change?

Use the bar graph for Exercises 3 and 4.

3. Estimate the total number of animals at the shelter.
4. There are 3 times as many ____ as ____ at the animal shelter.

Use the double-bar graph for Exercises 5–7.

5. About how much more is a club level seat at stadium A than at stadium B?
6. Which type of seat is the closest in price at the two stadiums?
7. Describe one relationship between the ticket prices at stadium A and stadium B.

Use the line graph for Exercises 8 and 9.

8. Estimate the number of tickets sold during the week of the greatest sales.
9. Which one-week period of time saw the greatest change in sales?

Use the double-line graph for Exercises 10–12.

10. When was the support for the two candidates closest?
11. Estimate the difference in voter support for the two candidates five weeks before the election.
12. Describe the general trend(s) of voter support for the two candidates.
Use the circle graph for Exercises 13–15.

13. Which color is least represented in the ball playpen?
14. There are 500 balls in the playpen. How many are yellow?
15. Which two colors are approximately equally represented in the ball playpen?

16. The table shows the breakdown of Karim’s monthly budget of $100. Use the given data to make a graph. Explain why you chose that type of graph.

<table>
<thead>
<tr>
<th>Item/Activity</th>
<th>Spending ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clothing</td>
<td>35</td>
</tr>
<tr>
<td>Food</td>
<td>25</td>
</tr>
<tr>
<td>Entertainment</td>
<td>25</td>
</tr>
<tr>
<td>Other</td>
<td>15</td>
</tr>
</tbody>
</table>

**PRACTICE AND PROBLEM SOLVING**

Use the bar graph for Exercises 17 and 18.

17. Estimate the difference in population between the tribes with the largest and the smallest population.
18. Approximately what percent of the total population shown in the table is Cherokee?

**Ray’s Restaurant**

<table>
<thead>
<tr>
<th>Day of the week</th>
<th>Lunch</th>
<th>Dinner</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>250</td>
<td>150</td>
</tr>
<tr>
<td>M</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>T</td>
<td>150</td>
<td>250</td>
</tr>
<tr>
<td>W</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>T</td>
<td>125</td>
<td>175</td>
</tr>
<tr>
<td>F</td>
<td>225</td>
<td>150</td>
</tr>
<tr>
<td>S</td>
<td>200</td>
<td>250</td>
</tr>
</tbody>
</table>

Use the double bar graph for Exercises 19–21.

19. On what day did Ray do the most overall business?
20. On what day did Ray have the busiest lunch?
21. On Sunday, about how many times as great was the number of dinner customers as the number of lunch customers?

Use the line graph for Exercises 22 and 23.

22. Between which two games did Marlon’s score increase the most?
23. Between which three games did Marlon’s score increase by about the same amount?
Use the double-line graph for Exercises 24–26.

24. What was the average value per share of Juan’s two stocks in July 2004?
25. Which stock’s value changed the most over any time period?
26. Describe the trend of the values of both stocks.

Use the circle graph for Exercises 27 and 28.

27. About what percent of the total number of cars are hopper cars?
28. About what percent of the total number of cars are gondola or tank cars?

29. The table shows the weight of twin babies at various times from birth to four weeks old. Use the given data to make a graph. Explain why you chose that type of graph.

Write bar, double-bar, line, double-line, or circle to indicate the type of graph that would best display the data described.

30. attendance at a carnival each year over a ten-year period
31. attendance at two different carnivals each year over a ten-year period
32. attendance at five different carnivals during the same year
33. attendance at a carnival by age group as it relates to total attendance
34. Critical Thinking Give an example of real-world data that would best be displayed by each type of graph: line graph, circle graph, double-bar graph.

35. This problem will prepare you for the Multi-Step Test Prep on page 710.

The first modern Olympic Games took place in 1896 in Athens, Greece. The circle graph shows the total number of medals won by several countries at the Olympic Games of 1896.

a. Which country won the most gold? Estimate the percent of the medals won by this country.

b. Which country won the second most medals? Estimate the percent of the medals won by this country.
36. **Write About It** Explain how you could use a line graph to make predictions.

37. Which type of graph would best display the contribution of each high school basketball player to the team, in terms of points scored?

- Bar graph
- Line graph
- Double-line graph
- Circle graph

38. At what age did Marianna have 75% more magazine subscriptions than she did at age 40?

- 25
- 30
- 35
- 45

39. **Short Response** The table shows the number of students in each algebra class. Make a graph to display the data. Explain why you chose that type of graph.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr. Abrams</td>
<td>34</td>
</tr>
<tr>
<td>Ms. Belle</td>
<td>29</td>
</tr>
<tr>
<td>Mr. Marvin</td>
<td>25</td>
</tr>
<tr>
<td>Ms. Swanson</td>
<td>27</td>
</tr>
</tbody>
</table>

40. On which trip were there more boys than girls?

41. A total of 60 people went to the museum. Estimate the number of girls who went to the museum.

42. Explain why it is not possible to determine whether fewer teachers went to the museum than to the zoo or the opera.

43. Find the domain and range for each relation and tell whether the relation is a function. *(Lesson 4-2)*

- \{(-3, 3), (-1, 1), (0, 0), (1, 1), (3, 3)\}

44. | x  | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

45. Triangle \(ABC\) has vertices on a coordinate plane as follows:

- \(A = (0, 5)\), \(B = (3, 0)\), \(C = (8, 3)\). Show that \(\triangle ABC\) is a right triangle. *(Lesson 5-8)*

46. \(24y\)

47. \(3x^2 + 6\)

48. \(4m - 18m^2 - 45m^3 + 120\)
Objectives
Create stem-and-leaf plots.
Create frequency tables and histograms.

Vocabulary
stem-and-leaf plot
frequency
data value
Stems are always consecutive numbers.
In Example 1B, a player has scores that start with 15, so there are no leaves in that row.

Why learn this?
Stem-and-leaf plots can be used to organize data, like the number of students in elective classes. (See Example 1.)

A stem-and-leaf plot arranges data by dividing each data value into two parts. This allows you to see each data value.

Key:
2|3 means 23

Making a Stem-and-Leaf Plot

A
The numbers of students in each of the elective classes at a school are given below. Use the data to make a stem-and-leaf plot.
24, 14, 12, 25, 32, 18, 23, 24, 9, 18, 34, 28, 24, 27

Number of Students in Elective Classes

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>2 4 8 8</td>
</tr>
<tr>
<td>2</td>
<td>3 4 4 5 7 8</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Key: 2|3 means 23

B
Marty's and Bill's scores for ten games of bowling are given below. Use the data to make a back-to-back stem-and-leaf plot.
Marty: 137, 149, 167, 134, 127, 143, 123, 168, 162
Bill: 129, 138, 141, 124, 139, 160, 149, 145, 128, 130

Bowling Scores

<table>
<thead>
<tr>
<th>Bowling Scores</th>
<th>Marty</th>
<th>Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 3 1</td>
<td>12</td>
<td>4 8 9</td>
</tr>
<tr>
<td>7 4</td>
<td>13</td>
<td>0 8 9</td>
</tr>
<tr>
<td>9 3</td>
<td>14</td>
<td>1 5 9</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 7 2</td>
<td>16</td>
<td>0</td>
</tr>
</tbody>
</table>

Key: [14][1 means 143]

1. The temperature in degrees Celsius for two weeks are given below. Use the data to make a stem-and-leaf plot.
7, 32, 34, 31, 26, 27, 23, 19, 22, 29, 30, 36, 35, 31
The **frequency** of a data value is the number of times it occurs. A **frequency table** shows the frequency of each data value. If the data is divided into intervals, the table shows the frequency of each interval.

### Example 2: Making a Frequency Table

The final scores for each golfer in a tournament are given below. Use the data to make a frequency table with intervals.

77, 71, 70, 82, 75, 76, 72, 70, 77, 74, 71, 75, 68, 72, 75, 74

**Step 1** Identify the least and greatest values.

The least value is 68. The greatest value is 82.

**Step 2** Divide the data into equal intervals.

For this data set, use an interval of 3.

**Step 3** List the intervals in the first column of the table. Count the number of data values in each interval and list the count in the last column. Give the table a title.

<table>
<thead>
<tr>
<th>Golf Tournament Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scores</strong></td>
</tr>
<tr>
<td>68–70</td>
</tr>
<tr>
<td>71–73</td>
</tr>
<tr>
<td>74–76</td>
</tr>
<tr>
<td>77–79</td>
</tr>
<tr>
<td>80–82</td>
</tr>
</tbody>
</table>

### Example 3: Making a Histogram

Use the frequency table in Example 2 to make a histogram.

**Step 1** Use the scale and interval from the frequency table.

**Step 2** Draw a bar for the number of scores in each interval.

All bars should be the same width. The bars should touch, but not overlap.

**Step 3** Title the graph and label the horizontal and vertical scales.

### Check It Out

2. The number of days of Maria's last 15 vacations are listed below. Use the data to make a frequency table with intervals.

4, 8, 6, 7, 5, 4, 10, 6, 7, 14, 12, 8, 10, 15, 12

A **histogram** is a bar graph used to display the frequency of data divided into equal intervals. The bars must be of equal width and should touch, but not overlap.

3. Make a histogram for the number of days of Maria's last 15 vacations.

4, 8, 6, 7, 5, 4, 10, 6, 7, 14, 12, 8, 10, 15, 12
Cumulative frequency shows the frequency of all data values less than or equal to a given value. You could just count the number of values, but if the data set has many values, you might lose track. Recording the data in a cumulative frequency table can help you keep track of the data values as you count.

**EXAMPLE 4**

**Making a Cumulative Frequency Table**

The heights in inches of the players on a school basketball team are given below.

72, 68, 71, 70, 73, 69, 79, 76, 72, 75, 72, 74, 68, 70, 69, 75, 72, 71, 73, 76

a. Use the data to make a cumulative frequency table.

- **Step 1** Choose intervals for the first column of the table.
- **Step 2** Record the frequency of values in each interval for the second column.
- **Step 3** Add the frequency of each interval to the frequencies of all the intervals before it. Put that number in the third column of the table.
- **Step 4** Title the table.

<table>
<thead>
<tr>
<th>Height (in.)</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>68–70</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>71–73</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>74–76</td>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>77–79</td>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>

b. How many players have heights under 74 in?

All heights under 74 in. are displayed in the first two rows of the table, so look at the cumulative frequency shown in the second row.

There are 14 players with heights under 74 in.

4. The number of vowels in each sentence of a short essay are listed below.

33, 36, 39, 37, 34, 35, 43, 35, 28, 32, 36, 35, 29, 40, 33, 41, 37

a. Use the data to make a cumulative frequency table.

b. How many sentences contain 35 vowels or fewer?

**THINK AND DISCUSS**

1. In a stem-and-leaf plot, the number of ______? ______ is always the same as the number of data values. (stems or leaves)

2. Explain how to make a histogram from a stem-and-leaf plot.

3. GET ORGANIZED Copy and complete the graphic organizer.

**Know it! Note**

Bar Graphs vs Histograms

How are they alike? How are they different?
GUIDED PRACTICE

1. **Vocabulary** A(n) ? is a data display that shows individual data values. (stem-and-leaf plot or histogram)

2. **Sports** The ages of professional basketball players at the time the players were recruited are given below. Use the data to make a stem-and-leaf plot.

3. **Weather** The average monthly rainfall for two cities (in inches) is given below. Use the data to make a back-to-back stem-and-leaf plot.

4. **Sports** The finishing times of runners in a 5K race, to the nearest minute, are given below. Use the data to make a frequency table with intervals.

5. **Biology** The breathing intervals of gray whales are given. Use the frequency table to make a histogram for the data.

6. The scores made by a group of eleventh-grade students on the mathematics portion of the SAT are given.
   a. Use the data to make a cumulative frequency table.
   b. How many students scored 650 or higher on the mathematics portion of the SAT?

**PRACTICE AND PROBLEM SOLVING**

7. The numbers of people who visited a park each day over two weeks during different seasons are given below. Use the data to make a back-to-back stem-and-leaf plot.
8. **Weather** The daily high temperatures in degrees Fahrenheit in a town during one month are given below. Use the data to make a stem-and-leaf plot.

<table>
<thead>
<tr>
<th>Daily High Temperatures (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>68 72 79 77 70 72 75 71 64 64</td>
</tr>
<tr>
<td>68 62 70 71 78 83 83 87 91 89</td>
</tr>
<tr>
<td>87 75 73 70 69 69 62 58 71 76</td>
</tr>
</tbody>
</table>

9. The overall GPAs of several high school seniors are given below. Use the data to make a frequency table with intervals.

<table>
<thead>
<tr>
<th>Overall GPAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.6 2.9 3.1 3.0 2.5 2.6 3.8 2.9</td>
</tr>
<tr>
<td>2.2 2.9 3.1 3.3 3.6 3.0 2.3 2.8 2.9</td>
</tr>
</tbody>
</table>

10. **Chemistry** The atomic masses of the nonmetal elements are given in the table. Use the frequency table to make a histogram for the data.

<table>
<thead>
<tr>
<th>Atomic Masses of Nonmetal Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval</td>
</tr>
<tr>
<td>0–49.9</td>
</tr>
<tr>
<td>50–99.9</td>
</tr>
<tr>
<td>100–149.9</td>
</tr>
<tr>
<td>150–199.9</td>
</tr>
<tr>
<td>200–249.9</td>
</tr>
</tbody>
</table>

11. The numbers of pretzels found in several samples of snack mix are given in the table.

<table>
<thead>
<tr>
<th>Numbers of Pretzels</th>
</tr>
</thead>
<tbody>
<tr>
<td>42 39 39 38 40</td>
</tr>
<tr>
<td>41 44 42 38 44</td>
</tr>
<tr>
<td>47 36 40 40 38</td>
</tr>
</tbody>
</table>

12. **Automobiles** The table shows gas mileage for the most economical cars in July 2004, including three hybrids.

<table>
<thead>
<tr>
<th>Gas Mileage of Economical Cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mileage in City (mi/gal)</td>
</tr>
<tr>
<td>Mileage on Highway (mi/gal)</td>
</tr>
</tbody>
</table>

Make a back-to-back stem-and-leaf plot for the data.

13. Damien’s math test scores are given in the table:

<table>
<thead>
<tr>
<th>Damien’s Math Test Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>75 84 68</td>
</tr>
<tr>
<td>72 59 88</td>
</tr>
<tr>
<td>72 77 81</td>
</tr>
<tr>
<td>84 60 70</td>
</tr>
</tbody>
</table>

a. Make a stem-and-leaf plot of Damien’s test scores.
b. Make a histogram of the test scores using intervals of 5.
c. Make a histogram of the test scores using intervals of 10.
d. Make a histogram of the test scores using intervals of 20.
e. How does the size of the interval affect the appearance of the histogram?
f. **Write About It** Which histogram makes Damien’s grades look highest? Explain.

14. /// **ERROR ANALYSIS** /// Two students made stem-and-leaf plots for the following data: 530, 545, 550, 555, 570. Which is incorrect? Explain the error.

```
A
<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>0</td>
</tr>
<tr>
<td>54</td>
<td>5</td>
</tr>
<tr>
<td>55</td>
<td>0 5</td>
</tr>
<tr>
<td>57</td>
<td>0</td>
</tr>
</tbody>
</table>

Key: 52|5 means 525

B
<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>0</td>
</tr>
<tr>
<td>54</td>
<td>5</td>
</tr>
<tr>
<td>55</td>
<td>0 5</td>
</tr>
<tr>
<td>56</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>0</td>
</tr>
</tbody>
</table>

Key: 52|5 means 525
```
15. This problem will prepare you for the Multi-Step Test Prep on page 710. The 2004 Olympic results for women’s weightlifting in the 48 kg weight class are 210, 205, 200, 190, 187.5, 182.5, 180, 177.5, 175, 172.5, 170, 167.5, and 165, measured in kilograms. Medals are awarded to the athletes who can lift the most weight.
   a. Create a frequency table beginning at 160 and using intervals of 10 kg.
   b. Create a histogram of the data.
   c. Tara Cunningham from the United States lifted 172.5 kg. Did she win a medal? How do you know?


   Ticket Sales (million $)
<table>
<thead>
<tr>
<th>Ticket Sales (million $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.9 19.7 18.8 13.5 13.1</td>
</tr>
<tr>
<td>11.2 10.2 7.5 6.1 5.1</td>
</tr>
</tbody>
</table>

17. **Critical Thinking** Margo’s homework assignment is to make a data display of some data she finds in a newspaper. She found a frequency table with the given intervals.

   Explain why Margo must be careful when drawing the bars of the histogram.

18. What data value occurs most often in the stem-and-leaf plot?

   **Stem|Leaves**
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

   **Key:** 3|2 means 3.2

19. The table shows the results of a survey about time spent on the Internet each month. Which statement is NOT supported by the data in the table?

   **Time Spent on the Internet per Month**
<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5–9</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>10–14</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>15–19</td>
<td>16</td>
<td>29</td>
</tr>
<tr>
<td>20–24</td>
<td>12</td>
<td>41</td>
</tr>
<tr>
<td>25–29</td>
<td>7</td>
<td>48</td>
</tr>
<tr>
<td>30–34</td>
<td>2</td>
<td>50</td>
</tr>
</tbody>
</table>

   - The interval of 30 to 34 h/mo has the lowest frequency.
   - More than half of those who responded spend more than 20 h/mo on the Internet.
   - Only four people responded that they spend less than 5 h/mo on the Internet.
   - Sixteen people responded that they spend less than 20 h/mo on the Internet.
20. The frequencies of starting salary ranges for college graduates are noted in the table. Which histogram best reflects the data?

<table>
<thead>
<tr>
<th>Salary Range ($)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000–29,000</td>
<td></td>
</tr>
<tr>
<td>30,000–39,000</td>
<td></td>
</tr>
<tr>
<td>40,000–49,000</td>
<td></td>
</tr>
<tr>
<td>50,000–59,000</td>
<td>1</td>
</tr>
</tbody>
</table>

**CHALLENGE AND EXTEND**

21. The cumulative frequencies of each interval have been given. Use this information to complete the frequency column.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>13–16</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>17–20</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>21–24</td>
<td></td>
<td>57</td>
</tr>
<tr>
<td>25–28</td>
<td></td>
<td>123</td>
</tr>
</tbody>
</table>

**SPIRAL REVIEW**

Solve each equation. (Lessons 2-3 and 2-4)

22. $19 = -2c + 5$

23. $4(m + 2) = -1.2$

24. $2(x - 3) + 7 = 3x - 9$

25. The U.S. standard railroad gauge is 56.5 inches, which is the distance between the track’s rails. Charles has a model train whose scale is 113:1. What is the distance between the rails on his model train track? (Lesson 2-6)

Use the circle graph for Exercises 26–28. (Lesson 10-1)

26. Which two types of gifts make up just over half of the donated gifts?

27. Which type of gift represents $\frac{1}{5}$ of the total donated gifts?

28. If there were 160 gifts donated, how many were books?
### 10-3 Data Distributions

**Objectives**
- Describe the central tendency of a data set.
- Create box-and-whisker plots.

**Vocabulary**
- mean
- median
- mode
- range
- outlier
- quartile
- interquartile range (IQR)
- box-and-whisker plot

**Who uses this?**
Sports analysts examine data distributions. (See Example 3.)

A *measure of central tendency* describes how data clusters around a value.

- The **mean** is the sum of the values in the set divided by the number of values in the set.
- The **median** is the middle value when the values are in numerical order, or the mean of the two middle values if there are an even number of values.
- The **mode** is the value or values that occur most often. There may be one mode or more than one mode. If no value occurs more often than another, we say the data set has no mode.

The **range** of a set of data is the difference between the least and greatest values in the set. The range describes the spread of the data.

#### Example 1
Find the mean, median, mode, and range of each data set.

**A**

The number of hours Isaac did homework on six days: 3, 8, 4, 6, 5, 4.

- Write the data in numerical order.
- Add all the values and divide by the number of values.
- There are an even number of values. Find the mean of the two middle values.
- 4 occurs more often than any other value.

- mean:
  \[ \frac{3 + 4 + 4 + 5 + 6 + 8}{6} = \frac{30}{6} = 5 \]

- median:
  3, 4, 4, 5, 6, 8
  The median is 4.5.

- mode: 4

- range: 8 – 3 = 5

**B**

The weight in pounds of Maria's five cats: 12, 14, 12, 16, 16.

- Write the data in numerical order.
- Add all the values and divide by the number of values.
- There are an odd number of values. Find the middle value.
- 12 and 16 both occur more often than any other value.

- mean:
  \[ \frac{12 + 12 + 14 + 16 + 16}{5} = \frac{70}{5} = 14 \]

- median:
  12, 12, 14, 16, 16
  The median is 14.

- mode: 12, 16

- range: 16 – 12 = 4

**Check it out!**
Find the mean, median, mode, and range of each data set.

1a. 8, 8, 14, 6
1b. 1, 5, 7, 2, 3
1c. 12, 18, 14, 17, 12, 18
A value that is very different from the other values in the set is called an **outlier**. In the data below, one value is much greater than the other values. This causes the mean to be greater than all of the other data values. In this case, either the median or mode would better describe the data.

---

**Example 2**

**Choosing a Measure of Central Tendency**

Niles scored 70, 74, 72, 71, 73 and 96 on six geography tests. Use the mean, median, and mode of his scores to answer each question.

- **mean** = 76
- **median** = 72.5
- **mode**: none

A Which value gives Niles' test average?

The average of Niles' scores is the mean, 76.

B Which value best describes Niles' scores? Explain.

The median score is the best description of Niles' six scores. Most of his scores were near 72.

The mean is higher than most of Niles' scores because he scored 96 on one test. Since there is no mode, it is not a good description of the data.

---

2. Josh scored 75, 75, 81, 84, and 85 on five tests. Use the mean, median, and mode of his scores to answer each question.

- **mean** = 80
- **median** = 81
- **mode** = 75

a. Which value describes the score Josh received most often?

b. Which value best describes Josh's scores? Explain.

---

**Helpful Hint**

Mathematically, any value that is 1.5(IQR) less than the first quartile or 1.5(IQR) greater than the third quartile is an outlier.

---

**Quartiles** divide a data set into four equal parts. Each quartile contains one-fourth of the values in the set. The **interquartile range (IQR)** is the difference between the upper and lower quartiles. The IQR represents the middle half of the data.

- **Range**: 9
- **IQR**: 4

A **box-and-whisker plot** can be used to show how the values in a data set are distributed. The minimum is the least value that is not an outlier. The maximum is the greatest value that is not an outlier. You need five values to make a box-and-whisker plot: the minimum, first quartile, median, third quartile, and maximum.


**EXAMPLE 3**  

**Sports Application**  

The numbers of runs scored by a softball team in 19 games are given. Use the data to make a box-and-whisker plot.

3, 4, 8, 12, 7, 5, 4, 12, 3, 9, 11, 4, 14, 8, 2, 10, 3, 10, 9

**Step 1** Order the data from least to greatest.

2, 3, 3, 3, 4, 4, 4, 5, 7, 8, 8, 9, 9, 10, 10, 11, 12, 12, 14

**Step 2** Identify the five needed values and determine whether there are any outliers.

- **Minimum**: 2
- **Q1**: 4
- **Q2**: 8
- **Q3**: 10
- **Maximum**: 14

IQR: 10 - 4 = 6 → 1.5(6) = 9  
4 - 9 = -5  
10 + 9 = 19

No values are less than -5 or greater than 19, so there are no outliers.

**Step 3** Draw a number line and plot a point above each of the five values you just identified. Draw a box through the lower and upper quartiles and a vertical line through the median. Draw lines from the box to the lower and upper extremes. (These are the whiskers.)

Half of the scores are between 4 and 10 runs per game. One-fourth of the scores are between 2 and 4. The greatest score earned by this team is 14.

3. Use the data to make a box-and-whisker plot.

13, 14, 18, 13, 12, 17, 15, 12, 13, 19, 11, 14, 14, 18, 22, 23

**THINK AND DISCUSS**

1. Explain when the median is a value in the data set.

2. GET ORGANIZED Copy and complete the graphic organizer. Tell which measure of central tendency answers each question.

<table>
<thead>
<tr>
<th>Measures of Central Tendency</th>
<th>Used to Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td></td>
</tr>
<tr>
<td>What is the average?</td>
<td></td>
</tr>
<tr>
<td>What is the halfway point of the data?</td>
<td></td>
</tr>
<tr>
<td>What is the most common value?</td>
<td></td>
</tr>
</tbody>
</table>

696  Chapter 10 Data Analysis and Probability
GUIDED PRACTICE

1. **Vocabulary** What is the difference between the **range** and the **interquartile range**?

Find the mean, median, mode, and range of each data set.

2. 85, 83, 85, 82
3. 12, 22, 33, 44, 44
4. 10, 26, 25, 10, 22, 25, 20
5. 71, 75, 78, 78, 80, 85, 86

6. The distance between five students' homes and the school are 3, 2, 2, 2, and 15 miles. Use the mean, median, and mode of the distances to answer each question.

   - mean = 2.8
   - median = 2
   - mode = 2

   a. Which value describes the distance between home and school that occurs most often?
   b. Which value best describes the distance between home and school? Explain.

Use the data to make a box-and-whisker plot.

7. 21, 31, 26, 24, 28, 26
8. 12, 13, 42, 62, 62, 82
9. 2, 1, 3, 2, 6, 2, 4
10. 104, 68, 90, 96, 101, 106, 95, 88

PRACTICE AND PROBLEM SOLVING

Find the mean, median, mode, and range of each data set.

11. 75, 63, 89, 91
12. 1, 2, 2, 2, 3, 3, 3, 4
13. 19, 25, 31, 19, 34, 22, 31, 34
14. 58, 58, 60, 60, 60, 61, 63
   Use the mean, median, and mode of Lamont's bowling scores to answer each question.
   a. Which value describes Lamont's average score?
   b. Which value best describes Lamont's scores? Explain.

Use the data to make a box-and-whisker plot.

16. 62, 63, 62, 64, 68, 62, 62
17. 85, 90, 81, 100, 92, 85
18. 1, 2, 3, 4, 5, 6, 7, 8
19. 17, 13, 19, 11, 17, 14, 11, 19, 12

Find the mean, median, mode, and range of each data set.

20. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
21. 5, 6, 6, 5, 5
22. 2, 1, 4, 3, 6, 5, 1, 2, 3, 4
23. 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1
24. 23, 25, 26, 25, 23
25. $-3$, $-3$, $-3$, $-2$, $-2$, $-1$
26. 1, 4, 9, 16, 25, 36
27. 51, 53, 51, 53, 51
28. 1, 0, 0, 0, 1, 1, 4

29. **Estimation** Estimate the mean of $16\frac{7}{8}$, $12\frac{1}{4}$, $22\frac{1}{10}$, $18\frac{5}{7}$, $19\frac{1}{3}$, and $13\frac{8}{11}$.

30. **Weather** The high temperatures in degrees Fahrenheit on 11 consecutive days were 68, 71, 75, 74, 77, 71, 73, 71, 72, 74, and 79. Find the mean, median, mode, and range of the temperatures. Then find the mean, median, mode, and range of the temperatures if the next day's temperature was 70°F. Describe the effect on the mean, median, mode, and range.
31. This problem will prepare you for the Multi-Step Test Prep on page 574.

In the 2004 Olympic games in Athens, the following results occurred for the men’s pole vault finals: 5.95, 5.90, 5.85, 5.80, 5.75, 5.75, 5.65, 5.65, 5.65, 5.55, 5.55, 5.55, 5.55. The results are heights in meters.

a. Find the mean, median, mode and range of this data set.

b. The gold medal was won by Timothy Mack of the United States. What was his height in the pole vault event?

c. Which measure of central tendency best describes the data set? Explain.

32. **Business** The salaries for eight people working for a small company are shown in the table. Determine the mean and median salaries. Which measure of central tendency best describes a typical salary of an employee at this company? Explain.

<table>
<thead>
<tr>
<th>Salaries ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000</td>
</tr>
<tr>
<td>25,000</td>
</tr>
<tr>
<td>20,000</td>
</tr>
<tr>
<td>25,000</td>
</tr>
<tr>
<td>30,000</td>
</tr>
<tr>
<td>35,000</td>
</tr>
<tr>
<td>100,000</td>
</tr>
</tbody>
</table>

33. Use the data to make a box-and-whisker plot.

34. 25, 28, 26, 16, 18, 15, 25, 28, 26, 16

35. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

36. **Sports** The table shows the attendance at 7 football games at Jefferson High School. Which measure of central tendency best shows the typical attendance at a football game?

<table>
<thead>
<tr>
<th>Attendance at Football Games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eagles vs. Bulldogs</td>
</tr>
<tr>
<td>Eagles vs. Panthers</td>
</tr>
<tr>
<td>Eagles vs. Coyotes</td>
</tr>
<tr>
<td>Eagles vs. Bears*</td>
</tr>
<tr>
<td>Eagles vs. Colts</td>
</tr>
<tr>
<td>Eagles vs. Mustangs</td>
</tr>
</tbody>
</table>

*Homecoming Game

37. **Write About It** Explain how an outlier with a large value will affect the mean. Explain how an outlier with a small value will affect the mean.

38. Allison has taken 5 tests worth 100 points each. Her scores are shown in the gradebook below. What score does she need on her next test to get an average of 90%?

<table>
<thead>
<tr>
<th>Student</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
<th>Test 5</th>
<th>Test 6</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allison</td>
<td>88</td>
<td>85</td>
<td>89</td>
<td>92</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brad</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tell whether each statement is *sometimes*, *always*, or *never* true.

39. The mean is a value in the data set.

40. The median is a value in the data set.

41. The mode is a value in the data set.

42. The mean is affected by including an outlier.

43. The mode is affected by including an outlier.
44. **Critical Thinking** Consider the given data set: 1, 2, 3, 5, 8, 13, 21.
   a. Find the mean of the given data set.
   b. What happens to the mean of the data set if every number is increased by 2?
   c. What happens to the mean of the data set if every number is multiplied by 2?

45. Which value must be represented on a box-and-whisker plot?
   - **A** Mean
   - **B** Median
   - **C** Mode
   - **D** Range

46. Which value must be a value in a data set?
   - **F** Mean
   - **G** Median
   - **H** Mode
   - **J** Range

47. Which of the following could be used to find the mean, median, mode, and range of a data set?
   - **A** Histogram
   - **B** Frequency table
   - **C** Stem-and-leaf plot
   - **D** Box-and-whisker plot

**CHALLENGE AND EXTEND**

48. List a set of data values with the following measures of central tendency: mean = 8, median = 7, mode = 6

49. For the box-and-whisker plot at right, how does the range of the lower half of the data differ from the range of the upper half of the data?

50. List a set of data values that can be represented by the box-and-whisker plot at right.

**SPIRAL REVIEW**

Find the slope of each line. Tell what rate the slope represents. *(Lesson 5-3)*

51. **Typing Speed**

52. **Diesel Fuel**

53. The length of a rectangle is one less than two times the width. The area is 15 yd². What are the dimensions of the rectangle? *(Lesson 9-6)*

54. The ages of the applicants for a driver’s license one day are shown in the table. Create a stem-and-leaf plot of the data. *(Lesson 10-2)*

<table>
<thead>
<tr>
<th>Ages of Applicants</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 16 21 16 16 17 35 16 18 17 16 16 23 18 30</td>
</tr>
</tbody>
</table>

10-3 Data Distributions 699
Technology LAB

Use Technology to Make Graphs

You can use a spreadsheet program to create bar graphs, line graphs, and circle graphs. You can also use a graphing calculator to make a box-and-whisker plot.

Use with Lesson 10-3

Activity 1

Many colors are used on the flags of the 50 United States. The table shows the number of flags that use each color. Use a spreadsheet program to make a bar graph to display the data.

<table>
<thead>
<tr>
<th>Color</th>
<th>Black</th>
<th>Blue</th>
<th>Brown</th>
<th>Gold</th>
<th>Green</th>
<th>Purple</th>
<th>Red</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>27</td>
<td>46</td>
<td>20</td>
<td>36</td>
<td>24</td>
<td>4</td>
<td>34</td>
<td>42</td>
</tr>
</tbody>
</table>

1. Enter the data from the table in the first two columns of the spreadsheet.

2. Select the cells containing the titles and the data.
   Then click the Chart Wizard icon, ![Chart Wizard Icon]. Click Column from the list on the left, and then choose the small picture of a vertical bar graph. Click Next.

3. The next screen shows the range of cells used to make the graph. Click Next.

4. Give the chart a title and enter titles for the x-axis and y-axis. Click the Legend tab, and then click the box next to Show Legend to turn off the key. (A key is needed when making a double-bar graph.) Click Next.

5. Click Finish to place the chart in the spreadsheet.
Try This

1. The table shows the average number of hours of sleep people at different ages get each night. Use a spreadsheet program to make a bar graph to display the data.

<table>
<thead>
<tr>
<th>Age (yr)</th>
<th>3–9</th>
<th>10–13</th>
<th>14–18</th>
<th>19–30</th>
<th>31–45</th>
<th>46–50</th>
<th>51+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleep (h)</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7.5</td>
<td>6</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Activity 2

Adrianne is a waitress at a restaurant. The amounts Adrianne made in tips during her last 15 shifts are listed below. Use a graphing calculator to make a box-and-whisker plot to display the data. Give the minimum, first quartile, median, third quartile, and maximum values.

$58, $63, $40, $44, $57, $59, $61, $53, $54, $58, $57, $57, $58, $58, $56

1. To make a list of the data, press \[ \text{STAT} \], select \text{Edit}, and enter the values in List 1 (L1). Press \[ \text{ENTER} \] after each value.

2. To use the \text{STAT PLOT} editor to set up the box-and-whisker plot, press \[ \text{2ND} \ Y= \], and then \[ \text{ENTER} \].

   Press \[ \text{ENTER} \] to select \text{Plot 1}.

3. Select \text{On}. Then use the arrow keys to choose the fifth type of graph, a box-and-whisker plot.
   \text{Xlist} should be L1 and \text{Freq:} should be 1.

4. Press \[ \text{ZOOM} \] and select 9: \text{ZoomStat} to see the graph in the statistics window.

5. Use \[ \text{TRACE} \] and the arrow keys to move the cursor along the graph to the five important values: minimum (MinX), first quartile (Q1), median (MED), third quartile (Q3), and maximum (MaxX).

   minimum: 40
   first quartile: 54
   median: 57
   third quartile: 58
   maximum: 63

Try This

2. The average length in inches of the ten longest bones in the human body are listed. Use a graphing calculator to make a box-and-whisker plot to display the data. What are the minimum, first quartile, median, third quartile, and maximum values of the data set?

Graphs can be used to influence what people believe. The way data is displayed can influence how the data is interpreted.

**EXAMPLE 1: Misleading Bar Graphs**

The graph shows the size of tomatoes on plants that were treated with different fertilizers.

A Explain why the graph is misleading. The scale on the vertical axis begins at 80. This exaggerates the differences between the sizes of the bars.

B What might someone believe because of the graph? Someone might believe that the tomato treated with fertilizer D is much larger than the other tomatoes. It is only 4 grams larger than the tomato treated with fertilizer B.

1. Who might want to use the graph above? Explain.

**EXAMPLE 2: Misleading Line Graphs**

The graph shows the average price of gasoline in the U.S. in September.

A Explain why the graph is misleading. The intervals on the vertical axis are not equal.

B What might people be influenced to believe by the graph? Someone might believe that the price of gasoline increased the most between 1995 and 1997. However, the change between 1995 and 1997 was only $0.14/gal while the change between 1999 and 2001 was $0.17/gal.

2. Who might want to use the graph above? Explain.
A circle graph compares each category of a data set to the whole. When any category is not represented in the graph, it may appear that another category represents a greater percentage of the total than it should.

**Example 3** Misleading Circle Graphs

The graph shows what percent of the total votes were received by three candidates for student council president.

A. Explain why the graph is misleading.
   The sections of the graph do not add to 100%, so the votes for at least one of the candidates is not represented.

B. What might people be influenced to believe by the graph?
   Someone might believe that Smith won the election.

3. Who might want to use the graph above? Explain.

Statistics can be misleading because of the way the data is collected or the way the results are reported. A random sample is a good way to collect unbiased data. In a random sample, all members of the group being surveyed have an equal chance of being selected.

**Example 4** Misleading Statistics

A researcher surveys people leaving a basketball game about what they like to watch on TV. Explain why the following statement is misleading: “80% of people like to watch sports on TV.”

The sample is biased because people who attend sporting events are more likely to watch sports on TV than people who watch TV but do not attend sporting events.

4. A researcher asks 4 people if they have seasonal allergies. Three people respond yes. Explain why the following statement is misleading: “75% of people have seasonal allergies.”

**Think and Discuss**

1. Give an example of a situation in which someone might intentionally try to make a graph misleading.

2. GET ORGANIZED Copy and complete the graphic organizer. Add more boxes if needed.
GUIDED PRACTICE

1. Vocabulary Explain in your own words what the term *random sample* means.

2. The graph shows the average salaries of employees at three companies.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

3. The graph shows hotel occupancy in San Francisco over four years.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

4. The graph shows the nutritional information for a granola bar.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

5. Three students were surveyed about their favorite teacher. Two students answer Mr. Gregory, and one answers Mr. Blaine. Explain why the following statement is misleading: “Mr. Gregory is the favorite teacher of a majority of the students.”

6. A researcher surveys people at a shopping mall about whether they favor enlarging the size of the mall parking lot. Explain why the following statement is misleading: “85% of the community is in favor of enlarging the parking lot.”
7. The graph shows the median rent for men and women in a metropolitan area.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

8. The graph shows the export prices of Colombian arabica coffee over nine years.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

10. A college math course has one section with 240 students and 8 sections with 30 students. Explain why the following statement is misleading: “The average class size for the course is 53 students.”

11. This problem will prepare you for the Multi-Step Test Prep on page 710.
    The table shows scores from the women’s gymnastics finals in the floor exercise at the 2004 Summer Olympic Games.
    a. Find the average score for the women in the finals.
    b. Why would it be misleading to say that this value is the average for women in the floor exercise?
    c. Make a graph for this data that could convince someone that the difference between the first place score and the eighth place score was very small.
12. **ERROR ANALYSIS** The graph shows the population of a city over time. Which conclusion is incorrect? Explain why the conclusion is incorrect and how the graph was misleading.

![Graph of city population over time]

**A** The population has grown rapidly over the past 20 years.  
**B** The population has grown slowly over the past 20 years.

13. The table shows the average connection speeds of some broadband Internet service providers.

<table>
<thead>
<tr>
<th>Provider</th>
<th>Connection Speed (Kbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speedy Online</td>
<td>954</td>
</tr>
<tr>
<td>TelQuick</td>
<td>914</td>
</tr>
<tr>
<td>Alacrity</td>
<td>858</td>
</tr>
</tbody>
</table>

a. Construct a display that suggests that Speedy Online is much faster than the other services.

b. Construct a display that suggests that all of the services offer about the same connection speeds.

c. **Write About It** Where might you expect to see your graph from part b? Explain.

14. **Critical Thinking** Explain how a graph can show truthful data but still be misleading.

15. What might someone be influenced to believe because of the graph?

- **A** The measles vaccine was introduced when the mortality rate was at its highest.
- **B** The measles vaccine was unnecessary.
- **C** The measles vaccine dramatically decreased the mortality rate.
- **D** The measles vaccine increased the mortality rate.

![Graph of United States measles mortality rates]

**United States Measles Mortality Rates**

- Measles vaccine introduced 1963

Source: www.healthsentinel.com

16. The table shows the number of votes cast in the 2000 U.S. presidential election and in the 2002 French presidential election. What additional information is needed to determine whether the following statement is misleading?

"American voters are more likely to vote than French voters."

- **F** The number of candidates in each election
- **G** The legal voting age in France
- **H** The number of registered voters in the United States in 2000 and France in 2002
- **J** The number of polling locations in the United States in 2000 and France in 2002

<table>
<thead>
<tr>
<th>Country</th>
<th>Votes Cast</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>105,405,100</td>
</tr>
<tr>
<td>France</td>
<td>29,497,272</td>
</tr>
</tbody>
</table>

706 Chapter 10 Data Analysis and Probability
CHALLENGE AND EXTEND

17. Logic A fingerprint analyst is studying a fingerprint that was found in the chemistry lab. He reports that the fingerprint belongs to Dr. Arenson. Below are two questions the analyst was asked and the answers he gave.

<table>
<thead>
<tr>
<th>Question 1: What are the chances that the fingerprint belongs to someone else who has the same fingerprint as Dr. Arenson?</th>
<th>Question 2: What are the chances that the fingerprint was wrongly identified?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answer: One in several billion.</td>
<td>Answer: About 1 in 100.</td>
</tr>
</tbody>
</table>

a. What is the difference between the two questions?
b. What does the answer to question 1 lead you to believe?
c. Who do you think might have asked question 1?
d. What does the answer to question 2 lead you to believe?
e. Who do you think might have asked question 2?

18. History Graphs like the one at right were created by Florence Nightingale. Nightingale served as a nurse during the Crimean War and was concerned with the unsanitary conditions the soldiers lived in. Each “wedge” of the circle represents a month between April 1854 and March 1855.

<table>
<thead>
<tr>
<th>Month</th>
<th>April 1854</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>August</th>
<th>September</th>
<th>October</th>
<th>November</th>
<th>December</th>
</tr>
</thead>
<tbody>
<tr>
<td>Causes of Death</td>
<td>Wounds in battle</td>
<td>Other causes</td>
<td>Disease</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>April 1854 to March 1855</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What do you think Florence Nightingale wanted to show with this graph?
b. Who do you think Nightingale showed the graph to?

History (1820–1910) served as a nurse in the Crimean War. In 1854, she brought the first female nurses to military hospitals.

SPIRAL REVIEW

Write an inequality for each situation. (Lesson 3-1)

19. The maximum weight for a certain truck load is 1500 pounds.

20. Isaac’s research paper must be at least 12 pages.

21. A moving company will transport up to 20 boxes for no fee.

Solve each inequality and graph the solutions. (Lesson 3-4)

22. $2x - 3 < 7$

23. $3(t - 1) \geq -15$

24. $6 - n < 2n + 9$

25. The table shows the weight of a golden retriever at different ages. Choose a type of graph to display the given data. Make the graph, and explain why you chose that type of graph. (Lesson 10-1)

<table>
<thead>
<tr>
<th>Age (mo)</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (lb)</td>
<td>8</td>
<td>23</td>
<td>45</td>
<td>66</td>
</tr>
</tbody>
</table>
Sampling and Bias

If you wanted to collect data about a very large group of people, you would need to survey a smaller group. The large group that contains all the people you could survey is called a population. The smaller group is called a sample.

You have learned that a random sample is a good way of collecting data that is unbiased. There are different ways of selecting a random sample.

**Random Samples**

<table>
<thead>
<tr>
<th>TYPE</th>
<th>DEFINITION</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Random Sample</td>
<td>Every member of the population has an equal chance of being chosen for the sample.</td>
<td>The names of all students in your class are placed in a hat and three names are chosen at random.</td>
</tr>
<tr>
<td>Stratified Random Sample</td>
<td>The population is divided into similar groups. Then a simple random sample is chosen from each group.</td>
<td>Your class is divided into boys and girls and two students are chosen at random from each group.</td>
</tr>
<tr>
<td>Systematic Random Sample</td>
<td>A member of the population is chosen for the sample at a regular interval.</td>
<td>Every third student who comes into the classroom is chosen.</td>
</tr>
</tbody>
</table>

**Example 1**

In each situation, identify the population and the sample. Tell whether each sample is a simple, stratified, or systematic random sample.

**A**

For one week, the manager of a pet supplies store asks every tenth customer what brand of pet food they buy.

population: all customers at the pet supplies store during one week
sample: every tenth customer during that same week

The sample is systematic because one member of the population is chosen for the sample at a regular interval.

**B**

Every person who enters a theater one evening places their ticket stub in a bowl. The theater owner chooses five ticket stubs to award prizes.

population: all people who put their ticket stub in the bowl
sample: five ticket stubs chosen from the bowl

The sample is simple because every member of the population has the same chance of being chosen.

**C**

One student from each classroom at a school is chosen at random for a committee.

population: all students at a school
sample: one student from each classroom

The sample is stratified because the population is divided into similar groups and one member is chosen at random from each group.
Try This

1. Choose a topic to research. Describe the population, why you need to choose a sample, and the sample.

2. Choose whether to use a simple, stratified, or systematic random sample. Explain your choice and the process you would use to choose your sample.

A random sample is not biased because no part of the population is favored over another. In a biased sample, one or more parts of the population have an advantage for being chosen for the sample.

There are two main types of biased samples.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convenience Sample</td>
<td>Those members of the population that are easily accessed are chosen for the sample.</td>
<td>A reporter questions people he personally knows.</td>
</tr>
<tr>
<td>Voluntary Response Sample</td>
<td>Members of the population who want to participate make up the sample.</td>
<td>A reporter questions people who fill out comment cards and indicate that they would like to be contacted.</td>
</tr>
</tbody>
</table>

Example 2

In each situation, identify the population and the sample. Tell whether each sample is a convenience or voluntary response sample. Explain why the sample is biased.

A website asks visitors to complete a survey about internet usage.
- population: all visitors to the website
- sample: those visitors who choose to complete the survey
The sample is a voluntary response sample because visitors to the website chose whether to complete the survey. The sample is biased because those visitors who choose to complete the survey may not be representative of all visitors to the website.

A reporter asks people leaving a shopping center through one door about their shopping habits.
- population: all people at the shopping center
- sample: people who leave from one door at the shopping center
The sample is a convenience sample because the people leaving through the chosen door are easily accessed. The sample is biased because people leaving through another door do not have an opportunity to be chosen.

Try This

3. Chose a topic to research. Describe the population, why you need to choose a sample, and the sample.

4. Assume that you want an unbiased sample. Choose whether to use a simple, stratified, or systematic random sample. Explain your choice and the process you would use to choose your sample.

5. Describe how a biased sample could be chosen for this same situation.
Quiz for Lessons 10-1 Through 10-4

**10-1 Organizing and Displaying Data**

Use the circle graph for Problems 1–3.

1. Which material represents over 50% of Don’s recyclables?
2. How many pounds of materials does Don recycle?
3. What percent of Don’s recyclables are glass?

4. The table shows the total proceeds for a fund-raiser at various times during the day. Choose a type of graph to display the given data. Make the graph, and explain why you chose that type of graph.

<table>
<thead>
<tr>
<th>Time</th>
<th>Total Proceeds (thousand $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3:00 P.M.</td>
<td>1.5</td>
</tr>
<tr>
<td>4:00 P.M.</td>
<td>2</td>
</tr>
<tr>
<td>5:00 P.M.</td>
<td>4</td>
</tr>
<tr>
<td>6:00 P.M.</td>
<td>6.5</td>
</tr>
<tr>
<td>7:00 P.M.</td>
<td>8</td>
</tr>
<tr>
<td>8:00 P.M.</td>
<td>9.5</td>
</tr>
</tbody>
</table>

**10-2 Frequency and Histograms**

5. The Miller family is going to buy a new car. The approximate prices of the cars they are considering are given. Use the data to make a stem-and-leaf plot.

$24,000, $28,000, $26,000, $32,000, $30,000, $41,000, $27,000, $22,000, $26,000, $33,000

6. The number of people at a caterer’s last 12 parties are given below.

16, 18, 17, 19, 15, 25, 18, 17, 18, 16, 17, 19

   a. Use the data to make a frequency table with intervals.
   b. Use your frequency table from part a to make a histogram.

**10-3 Data Distributions**

7. The daily high temperatures on 14 consecutive days in one city were 59°F, 49°F, 48°F, 46°F, 47°F, 51°F, 49°F, 43°F, 45°F, 52°F, 51°F, 51°F, 51°F, and 38°F.

   a. Find the mean, median, and mode of the temperatures.
   b. Which value describes the average high temperature for the 14 days?
   c. Which value best describes the high temperatures? Explain.

8. Use the temperature data above to make a box-and-whisker plot.

**10-4 Misleading Graphs and Statistics**

9. The graph shows the value of a company’s stock over time. Explain why the graph is misleading. What might people believe because of the graph? Who might want to use this graph?

10. The results of an online survey of 230 people showed that 92% of the population felt very comfortable using technology. Explain why this statistic is misleading.
Simulations

A simulation can be used to model an experiment that would be difficult or inconvenient to actually perform. In this lab, you will conduct simulations.

Use with Lesson 10-5

Activity

The local movie theater is offering an opportunity for customers to win a free night at the movies. To win, you must collect six different letters to spell CINEMA. Each movie ticket sold during this promotion will have one of the six letters stamped on the back of the ticket. An equal number of tickets will be stamped with each of the letters.

1. Since there are six different letters that appear on the tickets an equal number of times, you can use a number cube to simulate collecting the six letters. Each of the numbers on the number cube will represent a letter. Each roll of the number cube will represent purchasing one movie ticket, and the number rolled will represent the letter stamped on the ticket.

2. The table shows the results of rolling the number cube until each number has been rolled once.

   a. Based on the results shown in the table, how many rolls did it take to get all six numbers?
   b. Based on the results in the table, how many movie tickets would you have to buy to get all six letters? If you purchased this number of tickets, would you be sure to win? Explain.

Try This

1. Repeat the simulation four more times and record the results.
2. Find the average number of rolls from all five simulations
   \[
   \text{average number of rolls} = \frac{\text{total number of rolls from 5 simulations}}{5}
   \]
3. Based on your answer to Problem 2, how many movie tickets would you have to buy to get all six letters? Is this number different from the answer you gave based on the results in the table above?
4. Would any of your answers have been different if you had used a different correspondence between the numbers and letters? Explain.
10-5 Experimental Probability

Objectives
Determine the experimental probability of an event.
Use experimental probability to make predictions.

Vocabulary
experiment
trial
outcome
sample space
event
probability
experimental probability
prediction

Why learn this?
Experimental probability can be used by manufacturers for quality control. (See Example 4.)

An experiment is an activity involving chance. Each repetition or observation of an experiment is a trial, and each possible result is an outcome. The sample space of an experiment is the set of all possible outcomes.

Experiment | Rolling a number cube | Tossing a coin | Spinning a game spinner
---|---|---|---
Sample Space | \{1, 2, 3, 4, 5, 6\} | \{heads, tails\} | \{red, blue, green, yellow\}

EXAMPLE 1 Identifying Sample Spaces and Outcomes
Identify the sample space and the outcome shown for each experiment.

A tossing two coins
Sample space: \{HH, HT, TH, TT\}
Outcome shown: heads, tails (H, T)

B spinning a game spinner
Sample space: \{yellow, red, blue, green\}
Outcome shown: green

1. Identify the sample space and the outcome shown for the experiment: rolling a number cube.

An event is an outcome or set of outcomes in an experiment. Probability is the measure of how likely an event is to occur. Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.

Impossible | As likely as not | Certain
---|---|---
0% | 50% | 100%

Events with a probability of 0% never happen.
Events with a probability of 50% have the same chance of happening as not.
Events with a probability of 100% always happen.
**Example 2**

**Estimating the Likelihood of an Event**

Write *impossible, unlikely, as likely as not, likely, or certain* to describe each event.

**A**

There are 31 days in August.
August always has 31 days. This event is *certain*.

**B**

Carlos correctly guesses a number between 1 and 1000.
Carlos must pick one outcome out of 1000 possible outcomes. This event is *unlikely*.

**C**

A coin lands heads up.
Heads is one of two possible outcomes. This event is *as likely as not*.

**D**

Cecilia rolls a 10 on a standard number cube.
A standard number cube is numbered 1 through 6. This event is *impossible*.

2. Write *impossible, unlikely, as likely as not, likely, or certain* to describe the event: Anthony rolls a number less than 7 on a standard number cube.

You can estimate the probability of an event by performing an experiment. The **experimental probability** of an event is the ratio of the number of times the event occurs to the number of trials. The more trials performed, the more accurate the estimate will be.

**Experimental Probability**

\[
\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}
\]

**Example 3**

**Finding Experimental Probability**

An experiment consists of spinning a spinner. Use the results in the table to find the experimental probability of each event.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>7</td>
</tr>
<tr>
<td>Blue</td>
<td>8</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
</tr>
</tbody>
</table>

**A** spinner lands on blue

\[
\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{8}{7+8+5} = \frac{8}{20} = \frac{2}{5}
\]

**B** spinner does not land on green

When the spinner does not land on green, it must land on red or blue.

\[
\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{7+8}{7+8+5} = \frac{15}{20} = \frac{3}{4}
\]
Use the information in Example 3 to find the experimental probability of each event.

3a. spinner lands on red
3b. spinner does not land on red

You can use experimental probability to make predictions. A prediction is an estimate or guess about something that has not yet happened.

**Example 4**

**Quality Control Application**

A manufacturer inspects 800 light bulbs and finds that 796 of them have no defects.

A What is the experimental probability that a light bulb chosen at random has no defects?

Find the experimental probability that a light bulb has no defects.

\[
\text{number of times the event occurs} = \frac{796}{800} = 0.995
\]

The experimental probability that a light bulb has no defects is 99.5%.

B The manufacturer sent a shipment of 2400 light bulbs to a retail store. Predict the number of light bulbs in the shipment that are likely to have no defects.

Find 99.5% of 2400.

\[0.995(2400) = 2388\]

The manufacturer predicts that 2388 light bulbs have no defects.

4. A manufacturer inspects 1500 electric toothbrush motors and finds 1497 to have no defects.

a. What is the experimental probability that a motor chosen at random will have no defects?

b. There are 35,000 motors in a warehouse. Predict the number of motors that are likely to have no defects.

**Think and Discuss**

1. Explain the difference between an outcome and an event.

2. Is the experimental probability of an event always the same? Explain why or why not.

3. **Get Organized** Copy and complete the graphic organizer. In each box, write an example of an event that has the given likelihood.
1. **Vocabulary**  Give an example of an event that has two possible outcomes.

2. Identify the sample space and the outcome shown for each experiment.
   - rolling a number cube
   - spinning a spinner
   - tossing 3 coins

3. Write impossible, unlikely, as likely as not, likely, or certain to describe each event.
   - Peter was born in January. Thomas was born in June. Peter and Thomas have the same birthday.
   - The football team won 9 of its last 10 games. The team will win the next game.
   - A board game has a rule that if you roll the game cube and get a 6, you get an extra turn. You get an extra turn on your first roll.

4. An experiment consists of rolling a number cube. Use the results in the table to find the experimental probability of each event.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

5. Sports  One game of bowling consists of ten frames. Elyse usually rolls 3 strikes in each game.
   - a. What is the experimental probability that Elyse will roll a strike on any frame?
   - b. Predict the number of strikes Elyse will throw in 18 games.
Write impossible, unlikely, as likely as not, likely, or certain to describe each event.

15. Marlo purchased a new pair of shoes. She takes one shoe out of the box. The shoe is for the left foot.

16. Sam takes the bus to school. The bus came late twice in the last two weeks. The bus will be late today.

17. Tammy dropped two quarters on the floor. At least one of them lands heads up.

An experiment consists of randomly choosing a marble from a bag. Use the results in the table to find the experimental probability of each event.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>4</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
</tr>
<tr>
<td>Green</td>
<td>6</td>
</tr>
<tr>
<td>Yellow</td>
<td>9</td>
</tr>
</tbody>
</table>

18. choosing a yellow marble

19. choosing a blue marble

20. not choosing a green marble

21. **Sports** A ski lodge inspects 80 skis and finds 4 to be defective.
   a. What is the experimental probability that a ski chosen at random will be defective?
   b. The lodge has 420 skis. Predict the number of skis that are likely to be defective.

22. The table shows the results of a survey asking students the season of their birthday. What is the experimental probability that a student has a birthday during the summer?

23. You and your friend can either go swimming or to a movie on Thursday. The weather forecast says there is a 70% chance of rain on Thursday. Should you plan on going swimming or to a movie? Explain.

24. **Critical Thinking** Tell why it is important to repeat an experiment many times.

25. **Write About It** Explain what it means for an event to have a 50-50 chance of happening.

26. How many outcomes are in the sample space for an experiment consisting of rolling two standard number cubes?

27. **Estimation** A manufacturing company produced 986 units in one day. Of those, 9 units were found to be defective. Estimate the experimental probability that a unit produced that day was defective. Then predict approximately how many units will be defective when 5680 units are produced in one week.

28. This problem will prepare you for the Multi-Step Test Prep on page 744.

In a standard deck of cards, there are 13 cards in each of four suits: hearts, diamonds, clubs, and spades. The hearts and diamonds are red and the clubs and spades are black. Ricardo randomly drew cards from a standard deck of 52 cards. The table shows the results.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hearts</td>
<td>7</td>
</tr>
<tr>
<td>Diamonds</td>
<td>7</td>
</tr>
<tr>
<td>Clubs</td>
<td>8</td>
</tr>
<tr>
<td>Spades</td>
<td>6</td>
</tr>
</tbody>
</table>

   a. Find the experimental probability of drawing a club.
   b. Find the experimental probability of drawing a black suit.
29. Alex rolls two standard number cubes. What is the likelihood that the sum of the numbers is less than 4?
   A. Impossible   B. Unlikely   C. As likely as not   D. Likely

30. A community reported that \( \frac{2}{3} \) of the residents had a pet and \( \frac{1}{2} \) of the pet owners had a dog. If there are 84 residents in the community, how many residents are likely to have a dog?
   F. 14   G. 28   H. 42   J. 56

31. What is the probability that a number chosen at random from the list below will be a solution of the inequality \( 3x + 2 \leq 23 \)?
   \(-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\)
   A. \( \frac{17}{18} \)   B. \( \frac{8}{9} \)   C. \( \frac{1}{9} \)   D. \( \frac{1}{18} \)

32. Short Response
   A coin was tossed 50 times. It landed showing heads 6 more times than it landed showing tails. What is the experimental probability of the coin landing on heads? Show your work.

**CHALLENGE AND EXTEND**

A coin is tossed 3 times. Use the sample space for this experiment to describe each event below as impossible, unlikely, as likely as not, likely, or certain. Justify each answer.

33. At least 2 heads
34. 2 heads and 1 tail
35. 2 tails and 1 head
36. 3 tails
37. One coin is tossed 20 times.
   a. The experimental probability of the coin showing heads is 65%. How many times did the coin show tails?
   b. If the coin is tossed ten more times, how many more times must the coin land showing tails for the experimental probability of tails to be 50%?

**SPIRAL REVIEW**

38. A sales representative earns 4.5% commission on sales. Find the commission earned when the total sales are $124,000. (Lesson 2-9)
39. Estimate the tax on a $255 printer when the tax rate is 5.5%. (Lesson 2-9)

Compare the graph of each function to the graph of \( f(x) = x^2 \). (Lesson 9-4)
40. \( g(x) = \frac{1}{3}x^2 \)
41. \( g(x) = -x^2 \)
42. \( g(x) = x^2 - 12 \)

The data shows the number of books read by seven students over the summer:
5, 5, 14, 2, 5, 5, 6. (Lesson 10-3)
43. Give the mean, median, and mode of the data.
44. Which measure of central tendency best describes the data? Explain.
45. Create a box-and-whisker plot of the data.
Use Random Numbers

A calculator can be used to model an experiment that would be difficult or inconvenient to perform. To do this, you will use random numbers.

**Activity**

You can use a calculator to explore the experimental probability that at least 2 people in a group of 6 people were born in the same month. Assume that all months are equally likely to be a person's birth month.

1. Represent each month with an integer. Since there are 12 months, use the numbers 1–12.
   - To set your calculator up to generate random numbers, press `MATH`. Then use the arrow keys to highlight `PRB`. Select 5: `randInt()`. 
2. Now give the start number, 1, press `ENTER`, and give the end number, 12.
   - Each time you press `ENTER`, the calculator will return an integer from 1 to 12.
3. You are considering a group of 6 people. This means you need 6 random numbers.

<table>
<thead>
<tr>
<th>Person</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td>11</td>
<td>4</td>
<td>9</td>
<td>3</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Trial 2</td>
<td>3</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

In the first trial, the number 9 appears twice. This means that two people have a birthday in the ninth month, September.

In the second trial, no number appears more than once. This means that none of the people were born in the same month.

**Try This**

1. Repeat the experiment until you have 10 trials of the experiment. Count the number of trials in which a number appears more than once. Divide this number by the number of trials, 10, to find the experimental probability that at least 2 people in a group of 6 people will have the same birth month.

2. Gather the results from at least 100 trials of the experiment. (Either perform all of the trials yourself or combine data with your classmates.) Using your results, what is the experimental probability that at least 2 people in a group of 6 people will have the same birth month? Compare the results from 100 trials to the results of 10 trials.

3. How could you set up the experiment to find the experimental probability that at least 2 people in a group of 6 people will have the same birthday (same month and same date)?
Objectives
Determine the theoretical probability of an event.
Convert between probabilities and odds.

Vocabulary
equally likely
theoretical probability
fair
complement
odds

Why learn this?
Theoretical probability can be used to determine the likelihood of different weather conditions.
(See Example 2.)

When the outcomes in the sample space of an experiment have the same chance of occurring, the outcomes are said to be equally likely.

Theoretical Probability
An experiment in which all outcomes are equally likely is said to be fair. You can usually assume that experiments involving coins and number cubes are fair.

Example 1
Finding Theoretical Probability
An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

A rolling a 3

There is one 3 on a number cube.

Number of ways the event can occur
Total number of equally likely outcomes

= 1
6

= 0.16

= 16\%
An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

**B. rolling a number greater than 3**

\[
\text{number of ways the event can occur} = \frac{3}{6} = \frac{1}{2} \quad \text{There are 3 numbers greater than 3.}
\]

\[
= 0.5 = 50\%
\]

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

1a. rolling an even number

1b. rolling a multiple of 3

When you toss a coin, there are two possible outcomes, heads or tails. The table below shows the theoretical probabilities and experimental results of tossing a coin 10 times.

<table>
<thead>
<tr>
<th>Experimental Probability</th>
<th>P(heads)</th>
<th>P(tails)</th>
<th>P(heads) + P(tails)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\frac{3}{10})</td>
<td>(\frac{7}{10})</td>
<td>(\frac{3}{10} + \frac{7}{10} = \frac{10}{10} = 1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Theoretical Probability</th>
<th>P(heads)</th>
<th>P(tails)</th>
<th>P(heads) + P(tails)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1)</td>
</tr>
</tbody>
</table>

The sum of the probability of heads and the probability of tails is 1, or 100%. This is because it is certain that one of the two outcomes will always occur.

\[P(\text{event happening}) + P(\text{event not happening}) = 1\]

The **complement** of an event is all the outcomes in the sample space that are not included in the event. The sum of the probabilities of an event and its complement is 1, or 100%, because the event will either happen or not happen.

\[P(\text{event}) + P(\text{complement of event}) = 1\]

**EXAMPLE 2**

**Finding Probability by Using the Complement**

The weather forecaster predicts a 20% chance of snow. What is the probability that it will not snow?

\[P(\text{snow}) + P(\text{not snow}) = 100\% \quad \text{Either it will snow or it will not snow.} \]

\[20\% + P(\text{not snow}) = 100\% \]

\[-20\% \quad \text{Subtract 20\% from both sides.} \]

\[P(\text{not snow}) = 80\%\]

2. A jar has green, blue, purple, and white marbles. The probability of choosing a green marble is 0.2, the probability of choosing blue is 0.3, the probability of choosing purple is 0.1. What is the probability of choosing white?
Odds are another way to express the likelihood of an event. The odds in favor of an event describe the likelihood that the event will occur. The odds against an event describe the likelihood that the event will not occur.

Odds are usually written with a colon in the form $a:b$, but can also be written as $a$ to $b$ or $\frac{a}{b}$.

### Odds in Favor of an Event

$$\text{odds in favor} = \frac{\text{number of ways an event can happen}}{\text{number of ways an event can fail to happen}} = \frac{a}{b}$$

### Odds Against an Event

$$\text{odds against} = \frac{\text{number of ways an event can fail to happen}}{\text{number of ways an event can happen}} = \frac{b}{a}$$

- $a$ represents the number of ways an event can occur.
- $b$ represents the number of ways an event can fail to occur.

The two numbers given as the odds will add up to the total number of possible outcomes. You can use this relationship to convert between odds and probabilities.

#### Example 3

**Converting Between Odds and Probabilities**

**A** The probability of choosing a red card from a standard deck of playing cards is 50%. What are the odds of choosing a red card?

*The probability of choosing a red card is 50%, so the probability of not drawing a red card is $100\% - 50\% = 50\%$.\*

Odds in favor = 50:50, or 1:1

The odds in favor of choosing a red card are 1:1.

**B** The odds against choosing a green marble from a bag are 5:3. What is the probability of choosing a green marble?

*The odds against green are 5:3, so the odds in favor of green are 3:5. This means there are 3 favorable outcomes and 5 unfavorable outcomes for a total of 8 possible outcomes.*

$$\frac{\text{number of ways event can happen}}{\text{total possible outcomes}} = \frac{3}{8}$$

The probability of choosing a green marble is $\frac{3}{8}$.

3. The odds in favor of winning a free drink are 1:24. What is the probability of winning a free drink?
1. **Vocabulary** All of the outcomes in the sample space that are not included in the event are called the \_\_?\. (theoretical probability, complement, or odds)

Find the theoretical probability of each outcome.

2. rolling a number divisible by 3 on a number cube
3. flipping 2 coins and both landing with tails showing
4. randomly choosing the letter S from the letters in STARS
5. rolling a prime number on a number cube

6. A spinner is green, red, and blue. The probability that a spinner will land on green is 15% and red is 35%. What is the probability the spinner will land on blue?
7. The probability of choosing a red marble from a bag is \(\frac{1}{3}\). What is the probability of not choosing a red marble?
8. You have a \(\frac{1}{30}\) chance of winning. What is the probability you will not win?
9. There is a \(\frac{1}{10}\) chance that you will be chosen as class representative. What is the probability that you will not be chosen?
10. The odds against a spinner landing on blue are 3:1. What is the probability of landing on blue?
11. The probability of choosing an ace from a deck of cards is \(\frac{1}{13}\). What are the odds of choosing an ace?
12. The probability of not winning a game is 80%. What are the odds of winning?
13. The odds in favor of a spinner landing on blue are 1:3. What is the probability of landing on blue?
PRACTICE AND PROBLEM SOLVING

Find the theoretical probability of each outcome.

14. rolling a 5 on a number cube
15. flipping 2 coins and 1 landing with heads showing, the other with tails showing
16. randomly choosing a blue marble from a bag of 5 blue marbles, 8 red marbles, and 7 yellow marbles
17. The probability of a spinner landing on yellow is $\frac{4}{9}$. What is the probability of it not landing on yellow?
18. There is a 3% probability of winning a game. Find the probability of not winning the game.
19. There is a 15% chance it will snow and a 15% chance it will rain. What is the probability that it will neither snow nor rain?
20. The odds against winning a contest are 99 : 1. What is the probability of not winning the contest?
21. The odds of choosing a white marble from a bag are 1 : 9. Find the probability of not choosing a white marble.
22. The probability of a spinner landing on green is 25%. What are the odds of the spinner not landing on green?

Use the spinner for Exercises 23–28.

23. $P(\text{red})$
24. $P(\text{green})$
25. $P(\text{not blue})$
26. odds in favor of yellow
27. odds against red
28. odds against green

29. Write About It A number cube is rolled. Which event has a greater theoretical probability: rolling a number less than 3 or rolling a number greater than three? Explain.

30. ERROR ANALYSIS The odds in favor of an event are 1 : 4. Two students converted these odds into the probability of the event NOT happening. Which is incorrect? Explain the error.

A

<table>
<thead>
<tr>
<th>Odds in favor of event</th>
<th>1:4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability that event</td>
<td>1</td>
</tr>
<tr>
<td>will not happen</td>
<td>5</td>
</tr>
</tbody>
</table>

B

<table>
<thead>
<tr>
<th>Odds in favor of event</th>
<th>1:4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability that event</td>
<td>4</td>
</tr>
<tr>
<td>will not happen</td>
<td>5</td>
</tr>
</tbody>
</table>

31. Critical Thinking The odds in favor of a certain event are the same as the odds against that event. What is the probability of the event occurring?

32. This problem will prepare you for the Multi-Step Test Prep on page 744.
Chutes and Ladders is a children’s game that uses a spinner with the numbers 1 through 6.
a. What is the probability of a spinning a 3?
b. What is the probability of spinning an odd number?
c. What is the probability of spinning a number that is less than or equal to 4?
33. **Write About It**  Explain how to convert odds to probability.

34. **Geometry**  The radius of each circle in the diagram is given. Find the probability that a point chosen at random will lie in the red area of the diagram.

35. Two coins are tossed. What is the probability that at least one of the coins lands with heads showing?
   - A) 25%
   - B) 33 1/3%
   - C) 50%
   - D) 75%

36. A standard number cube is rolled. Which has the greatest probability?
   - P(even)
   - P(less than 5)
   - P(not 2)
   - P(greater than 3)

37. Find the probability that a point chosen at random would fall in the yellow area.
   - A) 1/6
   - B) 2/9
   - C) 4/9
   - D) 2/3

**CHALLENGE AND EXTEND**

Use the results of 3 coin-tossing experiments in the table for Exercise 38.

38. Find the experimental probability for
   a. experiment 1.
   b. experiment 2.
   c. experiment 3.
   d. Find the theoretical probability of heads.
   e. **Write About It**  How do the experimental probabilities of each experiment compare to the theoretical probability?

**SPIRAL REVIEW**

39. The table shows the volume of water in an office water cooler over time. Find the rate of change for each time period. For which time period did the volume of water decrease at the slowest rate? *(Lesson 5-3)*

<table>
<thead>
<tr>
<th>Time of day</th>
<th>7:00 A.M.</th>
<th>9:00 A.M.</th>
<th>1:00 P.M.</th>
<th>4:00 P.M.</th>
<th>5:00 P.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (gal)</td>
<td>4.2</td>
<td>3.8</td>
<td>2.7</td>
<td>1.2</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Factor each trinomial. *(Lesson 8-4)*

40. $2x^2 + x - 21$
41. $4x^2 - 7x + 3$
42. $6x^2 + 23x + 20$

An experiment consists of choosing a card out of a deck and recording the results. Use the table to find the experimental probability of each event. *(Lesson 10-5)*

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hearts</td>
<td>2</td>
</tr>
<tr>
<td>Diamonds</td>
<td>6</td>
</tr>
<tr>
<td>Spades</td>
<td>5</td>
</tr>
<tr>
<td>Clubs</td>
<td>7</td>
</tr>
</tbody>
</table>
10-7 Independent and Dependent Events

Objectives
Find the probability of independent events.
Find the probability of dependent events.

Vocabulary
independent events
dependent events

Adam's teacher gives the class two lists of titles and asks each student to choose two of them to read. Adam can choose one title from each list or two titles from the same list.

Events are independent events if the occurrence of one event does not affect the probability of the other. Events are dependent events if the occurrence of one event does affect the probability of the other.

Example 1
Classifying Events as Independent or Dependent

Tell whether each set of events is independent or dependent. Explain your answer.

A. A dime lands heads up and a nickel lands heads up.
   The result of tossing a dime does not affect the result of tossing a nickel, so the events are independent.

B. You choose a colored game piece in a board game, and then your sister picks another color.
   Your sister cannot pick the same color you picked, and there are fewer game pieces for your sister to choose from after you choose, so the events are dependent.

Tell whether each set of events is independent or dependent. Explain your answer.

1a. A number cube lands showing an odd number. It is rolled a second time and lands showing 6.
1b. One student in your class is chosen for a project. Then another student in the class is chosen.
Suppose an experiment involves flipping two fair coins. The sample space of outcomes is shown by the tree diagram. Determine the theoretical probability of both coins landing heads up.

There are four possible outcomes in the sample space:

\[(H, H), (H, T), (T, H), (T, T)\]

Only one outcome includes both coins landing heads up.

The theoretical probability of both coins landing heads up is \(\frac{1}{4}\).

Now look back at the separate theoretical probabilities of each coin landing heads up. The theoretical probability in each case is \(\frac{1}{2}\). The product of these two probabilities is \(\frac{1}{4}\), the same probability shown by the tree diagram.

To determine the probability of two independent events, multiply the probabilities of the two events.

\[
P(A \text{ and } B) = P(A) \cdot P(B)
\]

**Example 2**

Finding the Probability of Independent Events

An experiment consists of randomly selecting a marble from a bag, replacing it, and then selecting another marble. The bag contains 7 blue marbles and 3 yellow marbles. What is the probability of selecting a yellow marble and then a blue marble?

Because the first marble is replaced after it is selected, the sample space for each selection is the same. The events are independent.

\[
P(\text{yellow, blue}) = P(\text{yellow}) \cdot P(\text{blue})
\]

\[
= \frac{3}{10} \cdot \frac{7}{10}
\]

The probability of selecting yellow is \(\frac{3}{10}\) and the probability of selecting blue is \(\frac{7}{10}\).

\[
= \frac{21}{100}
\]
When a person rolls 2 dice and they land showing the same number, we say the person rolled doubles. What is the probability of rolling doubles 3 times in a row?

The result of one roll does not affect any following rolls. The events are independent.

When you roll a pair of dice, there are 36 possible outcomes, six of which are doubles:

So, the probability of rolling doubles once is 
\[ P(\text{double}) = \frac{6}{36} = \frac{1}{6}. \]

\[ P(\text{double, double, double}) = P(\text{double}) \cdot P(\text{double}) \cdot P(\text{double}) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}. \]

2. An experiment consists of spinning the spinner twice. What is the probability of spinning two odd numbers?

Suppose an experiment involves drawing marbles from a bag. Determine the theoretical probability of drawing a blue marble and then drawing a second blue marble without replacing the first one.

The sample space for the second draw is not the same as the sample space for the first draw. There are fewer marbles in the bag for the second draw. This means the events are dependent.

Probability of drawing a red marble on the first draw = \( \frac{3}{9} = \frac{1}{3} \)

Probability of drawing a red marble on the second draw = \( \frac{2}{8} = \frac{1}{4} \)
To determine the probability of two dependent events, multiply the probability of the first event times the probability of the second event after the first event has occurred.

**Probability of Dependent Events**

If $A$ and $B$ are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$.

**EXAMPLE 3**

**Problem-Solving Application**

There are 7 pink flowers and 5 yellow flowers in a bunch. Jane selects a flower at random, and then Leah selects a flower at random from the remaining flowers. What is the probability that Jane selects a pink flower and Leah selects a yellow flower?

1. **Understand the Problem**

   The answer will be the probability that a yellow flower is chosen after a pink flower is chosen.

   List the important information:
   - Jane chooses a pink flower from 7 pink flowers and 5 yellow flowers.
   - Leah chooses a yellow flower from 6 pink flowers and 5 yellow flowers.

2. **Make a Plan**

   Draw a diagram.

   Flowers Jane can choose from
   - 7 pink
   - 5 yellow
   - 12 total

   Flowers Leah can choose from
   - 6 pink
   - 5 yellow
   - 11 total

   After Jane selects a flower, the sample space changes. So the events are dependent.

3. **Solve**

   $$P(\text{pink and yellow}) = P(\text{pink}) \cdot P(\text{yellow after pink})$$

   $$= \frac{7}{12} \cdot \frac{5}{11}$$

   $$= \frac{35}{132}$$

   The probability that Jane selects a pink flower and Leah selects a yellow flower is $\frac{35}{132}$.

4. **Look Back**

   Drawing a diagram helps you see how the sample space changes. This means the events are dependent, so you can use the formula for probability of dependent events.

**CHECK IT OUT!**

3. A bag has 10 red marbles, 12 white marbles, and 8 blue marbles. Two marbles are randomly drawn from the bag. What is the probability of drawing a blue marble and then a red marble?

10-7 Independent and Dependent Events 729
1. **Vocabulary** Two events are ______ if the occurrence of one event affects the probability of the other event. (independent or dependent)

Tell whether each set of events is independent or dependent. Explain your answers.

2. You draw a heart from a deck of cards and set it aside. Then you draw a club from the deck of cards.

3. You guess “true” on two true-false questions.

4. Your brother calls you on the phone. You hang up the phone, and then your neighbor calls you.

5. You order from a menu, and then your friend orders a different meal.

6. A doctors’ office schedules several patients. Then you make an appointment.

7. A coin is tossed three times. What is the probability of the coin landing heads up three times?

8. Seven cards are numbered from 1 to 7 and placed in a box. One card is selected at random and replaced. Another card is randomly selected. What is the probability of selecting two odd numbers?

9. Stacey rolls two number cubes. What is the probability that the sum of the numbers on the two number cubes is 7?

10. A number cube is rolled twice and a coin is tossed once. What is the probability of the coin landing heads up and the number cube landing with 2 showing both times?

11. A spinner with four equal sections of red, yellow, green, and blue is spun twice. What is the probability that it lands on yellow and then on green?
11. A bag contains 4 red marbles, 3 white marbles, and 6 blue marbles. What is the probability of randomly selecting a red marble, setting it aside, and then randomly selecting a white marble from the bag?

12. Seven cards are numbered from 1 to 7 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability of selecting two odd numbers?

13. There are 15 boys and 14 girls in a room. Two of them are selected at random to take a survey. What is the probability that the two people selected will be girls?

**PRACTICE AND PROBLEM SOLVING**

Tell whether each set of events is independent or dependent. Explain your answer.

14. The teacher randomly selects two students from the class.

15. You roll a 3 on a number cube and choose a 3 from a deck of cards.

16. A number cube is rolled three times. What is the probability of rolling three even numbers?

17. Ten cards are numbered from 1 to 10 and placed in a box. One card is selected at random and replaced. Another card is randomly selected. What is the probability of selecting two even numbers?

18. Stacey rolls a number cube and flips a coin. What is the probability that she rolls a 5 and the coin lands heads up?

19. A bag contains 5 red marbles, 3 white marbles, and 4 blue marbles. What is the probability of randomly selecting a red marble, setting it aside, and then randomly selecting another red marble from the bag?

20. Ten cards are numbered from 1 to 10 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability of selecting two even numbers?

21. A game has 6 colored playing pieces. They are red, yellow, green, blue, purple, and white. You and a friend pick your game piece without looking. What is the probability that your friend picks the blue piece and you pick the yellow piece?

22. **School** On a multiple-choice test, each question has 4 possible answers. A student does not know the answers to three questions, so the student guesses.

   a. What is the probability that the student gets all three questions wrong?
   b. What is the probability that the student gets all three questions right?

Tell whether each set of events is independent or dependent. Explain your answer.

23. Pick “Joe” from a box of names, replace it, and then pick “Craig.”

24. Pick “Joe” from a box of names, set it aside, and then pick “Craig.”

25. Roll a prime number on a number cube and get tails when flipping a coin.

26. Roll an even number, then an odd number, and then a 1 on a number cube.
28. This problem will prepare you for the Multi-Step Test Prep on page 744.
Yahtzee is a game that involves rolling five dice. On his or her turn, a player can roll up to three times to try to score points in various categories. Rolling a “Yahtzee” means rolling five of a kind, or five of the same number.
   a. Juan has rolled twice and has three 5's showing. He rolls the remaining two dice.
      What is the probability that both dice will land showing 5?
   b. Shauna has two 3's showing. She has one more roll with the remaining three dice.
      What is the probability that all three dice will land showing 3?
   c. Mike rolls all five number cubes and all of them land showing 6. What is the probability of getting five 6's in one roll?

29. A bag contains 3 red, 5 blue, and 2 white marbles.
   a. Find the probability of randomly picking a red marble, replacing it, and then picking a blue marble.
   b. Find the probability of randomly picking a red marble, setting it aside, and then picking a blue marble.
   c. Find the probability of randomly picking a red marble, replacing it, and then picking another red marble.
   d. Find the probability of randomly picking a red marble, setting it aside, and then picking another red marble.

30. Entertainment Joe and Maria are playing a board game. On each turn, the player rolls two number cubes. Both players have two turns remaining.
   a. Joe will win if he rolls double 6's on both turns. What is the probability that Joe will roll double 6's on both turns?
   b. Maria will win if she rolls 2 on the first turn and 12 on the second turn.
      What is the probability that Maria will roll 2 on the first turn and 12 on the second turn?
   c. Write About It Who has the better probability of winning? Explain.

31. Tamika has $2.50 in quarters in her pocket, including four state quarters. She reaches into her pocket and takes out two quarters. What is the probability that they are both state quarters?

32. Ten cards are numbered 1 through 10 and placed in a bag. You draw a card, set it aside, and draw another card. What is the probability that you will draw two numbers that are divisible by 3?

33. Critical Thinking What is the probability of a coin landing heads up on two flips if it lands tails up on the first flip? Explain.

34. Write About It Explain what it means for two events to be independent.

35. A number cube is rolled twice. What is the probability of getting a 2 on both rolls?
   A 1/3  B 1/4  C 1/9  D 1/36

36. In baseball, Julio averages 3 hits in every 10 at bats. What is the probability that he will get hits in both of his next two at bats?
   F 0.03  G 0.09  H 0.3  I 0.9

732  Chapter 10 Data Analysis and Probability
37. Two people from a group of 30 will be selected at random for a prize. Twenty people in the group are women. What is the probability that both people selected will be men?

$$\frac{3}{29}, \quad \frac{38}{87}, \quad \frac{56}{87}, \quad \frac{1}{10}$$

38. Ravi has 10 pairs of socks in a drawer, but none of the pairs are matched up. Each pair is a different color, and one pair is blue. Ravi has to pick his socks in the dark so he does not wake his brother. Which expression can be used to find the probability that Ravi will choose a blue sock and then the matching sock?

$$\frac{1}{20} + \frac{1}{20}, \quad \frac{1}{20} \cdot \frac{1}{20}, \quad \frac{1}{20} + \frac{1}{19}, \quad \frac{1}{19}$$

**CHALLENGE AND EXTEND**

39. **Basketball** Terrance has made 90% of the free throws he has attempted at basketball practice. What is the probability that he will make the next three free throws he attempts?
40. A number cube is rolled three times. What is the probability of rolling a 5 at least once?

**Geometry** Use the grid for Exercises 41–44.

On the grid at right, one small square represents 1% probability.
The pink area represents the probability that event \(A\) occurs.
The blue area represents the probability that event \(B\) occurs.
The area where the two colors overlap represents the probability that both events occur.
Use the grid to find each probability.

41. Event \(A\) occurs.
42. Event \(B\) occurs.
43. Event \(A\) occurs AND event \(B\) occurs.
44. Neither event \(A\) nor event \(B\) occurs.

**SPIRAL REVIEW**

45. A tennis player serves 2 aces (unreturned serves) for every 17 serves. If the player serves 204 times in the next match, how many serves would you expect to be aces? *(Lesson 2-6)*

Compare the graph of each function to the graph of \(f(x) = x^2\). *(Lesson 9-4)*

46. \(g(x) = 4x^2 - 1\)
47. \(g(x) = \frac{1}{5}x^2\)
48. \(g(x) = -x^2 + 3\)

Find the theoretical probability of each outcome. *(Lesson 10-6)*

49. Randomly selecting a blue marble out of a bag with 6 red and 9 blue marbles
50. Rolling a number less than 10 on a number cube
51. Randomly selecting A, E, I, O, or U from all letters of the alphabet
Compound Events

When two events cannot happen at the same time, they are called mutually exclusive events. When two events can happen at the same time, they are called inclusive events. You can use sample spaces to determine the probabilities of mutually exclusive events and inclusive events.

Activity 1

Use with Lesson 10-7

Try This

1. What is the total number of possible rolls?
2. What is the probability that the total will be 7?
3. What is the probability that the total will be 12?
4. What is the probability that the total will be 7 or 12?
5. What do you notice about the probabilities in Problems 2, 3, and 4?

Suppose you are three spaces away from Community Chest and eight spaces away from Chance.

6. What is the probability that on your next roll the total will be 3?
7. What is the probability that on your next roll the total will be 8?
8. What is the probability that on your next roll the total will be 3 or 8?
9. Complete the following statement:

The probability that one of two mutually exclusive events will occur is the \( \frac{?}{?} \) of the probabilities of the individual events.

In this lab, you discovered this rule: If \( A \) and \( B \) are mutually exclusive events, then \( P(A \text{ or } B) = P(A) + P(B) \).
Try This

10. Find the probability of Case 1, rolling at least one 1.
   a. What is the probability of rolling on the first die and any number on the second die?
   b. What is the probability of rolling any number on the first die and on the second die?
   c. What is the probability of rolling on the first die and on the second die?
   d. Parts a and b both include rolling on the first die and on the second die. You need to count this outcome only once, so subtract the probability from part c from the sum of parts a and b.

11. Find the probability of Case 2, rolling at least one 5. (Hint: Repeat the steps in Problem 1.)

12. Find the probability of Case 3, rolling a 1 and a 5.

13. The probability of rolling a small straight, is the probability of rolling or .
    Subtract the probability of rolling both and from the sum of the probabilities of rolling on one of the dice (Problem 1) and rolling on one of the dice (Problem 2).

14. Complete the following statement:
    The probability that one of two inclusive events will occur is the of the probabilities of the individual events the probability of the intersection of the two events.

In this lab, you discovered this rule:
If A and B are inclusive events, then 

10-7 Algebra Lab 735
Quiz for Lessons 10-5 Through 10-8

10-5  Experimental Probability
An experiment consists of pushing the random select button on a CD player for a disc with five tracks. Use the results in the table to find the experimental probability of each event.

1. Selecting track 4
2. Selecting track 3
3. Not selecting track 2
4. Selecting an odd numbered track
5. Selecting one of the first 3 tracks
6. Ms. Bleakman checks 32 papers and finds 2 with no name. What is the experimental probability that a paper chosen at random will have no name? Ms. Bleakman has the papers of 176 students. Predict the number of papers she will find with no name.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Track 1</td>
<td>4</td>
</tr>
<tr>
<td>Track 2</td>
<td>6</td>
</tr>
<tr>
<td>Track 3</td>
<td>2</td>
</tr>
<tr>
<td>Track 4</td>
<td>5</td>
</tr>
<tr>
<td>Track 5</td>
<td>3</td>
</tr>
</tbody>
</table>

10-6  Theoretical Probability
Find the theoretical probability of each outcome.

7. Landing on an odd number when spinning a spinner with 4 equal spaces marked 1, 4, 6, 8
8. Randomly selecting a vowel from all the letters in the word VOWELS
9. Picking a red marble out of a bag with 3 green, 5 blue, 2 red, and 6 yellow marbles
10. The probability of winning a certain prize is 5%. What are the odds of winning the prize?
11. The odds of choosing a winning ticket are 2 : 8. What is the probability of choosing a winning ticket?
12. The probability of snow is 70%. What are the odds of it not snowing?

10-7  Independent and Dependent Events
13. A physical education class has 12 boys and 18 girls. Each day, the teacher randomly selects a team captain. Assume that no student is absent. What is the probability that the team captain is a girl two days in a row?
14. After reading to a kindergarten class at the library, Tobey gives out stickers. He has 9 zoo animal stickers and 16 scratch-n-sniff stickers. If Tobey gives the stickers out at random, what is the probability that the first child gets a scratch-n-sniff sticker and the second child gets a zoo animal sticker?

10-8  Combinations and Permutations
15. Luisa is choosing a line-up of CDs to play from beginning to end at her party. She has 12 CDs from which to choose and time to play only 5. How many ways are there for Luisa to arrange the musical line-up?
16. Gavin is shopping for school clothes with his father. After trying on several shirts, Gavin has identified 8 shirts that he likes. His father says Gavin must narrow it down to 5 shirts. How many ways can Gavin choose 5 shirts from the 8 he likes?
Vocabulary

bar graph ................. 678  fair ......................... 720  outlier ..................... 695
box-and-whisker plot .... 695  frequency ...................... 688  permutation ............. 737
circle graph .............. 680  frequency table ........... 688  prediction ............... 715
combination ................ 737  histogram .................. 688  probability ............. 713
complement ............... 721  independent events ...... 726  quartile ..................... 695
compound event .......... 737  interquartile range (IQR) 695  random sample ........... 703
cumulative frequency ....... 689  line graph .................. 679  range ....................... 694
dependent events ......... 726  mean ......................... 694  sample space ........... 713
equally likely ............. 720  median ....................... 694  stem-and-leaf plot ...... 687
event ....................... 713  mode ......................... 694  theoretical probability 720
experiment ............... 713  odds ......................... 722  trial ....................... 713
experimental probability .. 714  outcome ..................... 713

Complete the sentences below with vocabulary words from the list above.
1. A(n) _______ is one possible result of an experiment.
2. The _______ is the difference between the upper and lower quartiles.
3. Two events are _______ if the occurrence of one event does not affect the probability of the other.

10-1 Organizing and Displaying Data (pp. 678–686)

**Example**
The circle graph shows the post-graduation plans for a high school's 500 graduating seniors.

- How many seniors plan to attend a four-year college?
  50% of 500
  0.50 × 500 = 250
  250 students plan to attend a four-year college.

**Exercises**
Use the double-bar graph for Exercises 4 and 5.

4. In which year did the same number of boys and girls participate?
5. How many more boys than girls participated during 2004?
10-2 Frequency and Histograms (pp. 687–693)

**EXAMPLE**

The lives (in hours) of each light bulb in a test are given below. Use the data to make a stem-and-leaf plot.

22, 20, 25, 21, 19, 21, 25, 21, 22, 18, 20, 29, 25, 26

Identify the least and greatest values. Divide the data into equal intervals.

**EXERCISES**

6. The weights of packages shipped by an online retailer are given below. Use the data to make a stem-and-leaf plot.

<table>
<thead>
<tr>
<th>Package Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 9 22 24 7 1 19 22 28 18 12</td>
</tr>
</tbody>
</table>

7. The numbers of people who attended two different plays are shown in the table below. Make a back-to-back stem-and-leaf plot.

<table>
<thead>
<tr>
<th>Play Attendance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comedy Camp</td>
</tr>
<tr>
<td>104 62 83 102</td>
</tr>
<tr>
<td>104 120 81</td>
</tr>
<tr>
<td>126 122</td>
</tr>
</tbody>
</table>

8. The capacities of the gas tanks on several new vehicles are shown below. Use the data to make a frequency table with intervals.

<table>
<thead>
<tr>
<th>Gas Tank Capacity (gal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 12 15 18 10 26 25 12 15 18 11</td>
</tr>
<tr>
<td>10 12 16 15 18 20 21</td>
</tr>
</tbody>
</table>

9. Use your frequency table from Exercise 9 to make a histogram.

10-3 Data Distributions (pp. 694–699)

**EXAMPLE**

Consider the following ages of the winners of an art contest: 13, 15, 14, 18, 12, 10, 11, 13. Find the mean and mode of the data.

mean:

\[
\frac{10 + 11 + 12 + 13 + 13 + 14 + 15 + 18}{8}
\]

= \(\frac{106}{8}\)

= 13.25

mode: 13 occurs most often.

**EXERCISES**

Find the mean, median, mode, and range of each data set.

10. Years of playing experience of the members of a musical ensemble:

5, 14, 25, 7, 8, 10, 12, 33, 12

Herman has five pairs of cowboy boots. The prices were $120, $137, $120, $145, and $482.

mean: $200.80 median: $137 mode: $120

11. Which value best describes the price Herman paid? Explain.

Use the data to make a box-and-whisker plot.

12. 25, 28, 2, 24, 28, 21, 18, 29, 31, 12, 6, 19, 27, 3
10-4 Misleading Graphs and Statistics (pp. 702–709)

**EXAMPLE**

- Explain why the graph is misleading.

The graph shows the cost of admission to an amusement park over 20 years.

The vertical axis begins at 71. This exaggerates the difference in the heights of the bars.

13. Explain why the graph is misleading.

14. What might someone believe because of the graph?

**EXERCISES**

13. Explain why the graph is misleading.

14. What might someone believe because of the graph?

10-5 Experimental Probability (pp. 713–718)

**EXAMPLE**

A manufacturer inspects 800 batteries and finds that 796 have no defects.

15. What is the experimental probability that a battery chosen at random has no defects?

16. There are 25,000 batteries in storage. How many batteries are likely to have no defects?

17. Another storage area holds 50,000 batteries. How many batteries are likely to have a defect?

**EXERCISES**

15. What is the experimental probability that a battery chosen at random has no defects?

16. There are 25,000 batteries in storage. How many batteries are likely to have no defects?

17. Another storage area holds 50,000 batteries. How many batteries are likely to have a defect?

10-6 Theoretical Probability (pp. 720–725)

**EXAMPLE**

A jar contains red, green, brown, and blue marbles. The probability of choosing red is 0.30, of choosing green is 0.20, and of choosing brown is 0.25. Find the probability of choosing blue.

\[
P(\text{blue}) + P(\text{not blue}) = 1
\]

\[
P(\text{blue}) + P(\text{red, green, or brown}) = 1
\]

\[
P(\text{blue}) + (0.30 + 0.20 + 0.25) = 1
\]

\[
P(\text{blue}) + 0.75 = 1
\]

\[
P(\text{blue}) = 0.25
\]

**EXERCISES**

18. Rolling a number less than 4 on a standard number cube

19. Randomly selecting a month that starts with "J" from all month names

20. Randomly selecting a vowel from the letters in EQUATION
Study Guide: Review 753

10-7 Independent and Dependent Events (pp. 726–733)

**Examples**

A hardware store shelf holds 12 cans of red paint, 4 cans of yellow paint, and 6 cans of black paint.

- Syd selects one can at random and replaces it. Then she selects another can at random. What is the probability that Syd selects a red can and then a yellow can?
  
  \[ P(\text{red, yellow}) = P(\text{red}) \cdot P(\text{yellow}) = \frac{12}{22} \cdot \frac{4}{22} = \frac{48}{484} = \frac{12}{121} \]

- Gene selects one can at random and then selects another can at random from the remaining cans. What is the probability that Gene selects two cans of black paint?
  
  \[ P(\text{black, black}) = P(\text{black}) \cdot P(\text{black after black}) = \frac{6}{22} \cdot \frac{5}{21} = \frac{30}{462} = \frac{5}{77} \]

**Exercises**

Tell whether each set of events is independent or dependent. Explain your answers.

21. A computer generates a random number and then generates another random number.
22. You roll two number cubes. One is a 6 and the other is a 1.
23. Two audience members are called to the stage.

A lottery machine contains different-colored balls. There are 64 green, 128 yellow, 1 golden, and 3 silver balls. Find the probability of each event.

24. A yellow ball is drawn and set aside. Then a green ball is drawn.
25. A golden ball is drawn and set aside. Then another golden ball is drawn.
26. A green ball is drawn and replaced. Then another green ball is drawn.

10-8 Combinations and Permutations (pp. 736–743)

**Examples**

A sporting goods store carries sweatshirts for 8 local high school football teams.

- How many different packages of 4 different high school sweatshirts are possible?
  
  Use a combination. The order does not matter.
  
  \[ \binom{8}{4} = \frac{8!}{4!(8-4)!} = \frac{8!}{4!4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 70 \]

- Three different high school sweatshirts will be hung in a row. How many displays are possible?
  
  Use a permutation. The order matters.
  
  \[ \binom{8}{3} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 336 \]

**Exercises**

27. A catering hall offers 4 different plates, 3 different silverware patterns, and 5 different types of glassware. How many place settings of one plate, one silverware pattern, and one type of glassware are possible?

Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

28. Shelly is making up a 7-digit phone number to use in a play. She can choose any digit from 0–9 but does not want to repeat a number. How many different phone numbers are possible?
29. A restaurant offers 12 different appetizers. How many ways can a group of friends share 3 different appetizers?
30. A group of 15 friends is at an amusement park. In how many ways can a group be chosen to ride in a four-person gondola?
1. The table shows the population of Oakville. Use the data to make a graph. Explain why you chose that type of graph.

2. Which ten-year period saw the greatest change in population?

3. Describe the trend in Oakville’s population.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>20,851</td>
</tr>
<tr>
<td>1980</td>
<td>14,229</td>
</tr>
<tr>
<td>1990</td>
<td>11,198</td>
</tr>
<tr>
<td>2000</td>
<td>9,579</td>
</tr>
</tbody>
</table>

The high temperatures in degrees Fahrenheit for two weeks are given: 64, 66, 63, 58, 59, 51, 54, 61, 62, 68, 70, 63, 63.

4. Use the data to make a stem-and-leaf plot.

5. Use the data to make a frequency table with intervals.

6. Use your frequency table from Problem 5 to make a histogram.

The lengths of statements during a town council meeting are given.

<table>
<thead>
<tr>
<th>Length of Statement (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6  25  12  14  2  13  38</td>
</tr>
<tr>
<td>22 21 14  3   8   5   17</td>
</tr>
</tbody>
</table>

7. Find the mean, median, mode, and range of the data.

8. Use the data to make a box-and-whisker plot.

A manufacturer inspects 500 watches and finds that 498 have no defects.

9. What is the experimental probability that a watch chosen at random has no defects?

10. There are 30,000 watches in a warehouse. Predict the number of watches that are likely to have no defects.

The graph shows how the money raised by a charity is spent.

11. Explain why the graph is misleading.

12. What might someone believe because of the graph?

13. Who might want to use this graph?

14. An experiment consists of pulling one card out of a bag that has 12 cards, each with a different month of the year printed on it. What is the probability that the month begins with A?

15. The odds of spinning red on a spinner are 2 : 7. What is the probability of not spinning red?

16. A bag has 14 red marbles and 10 white marbles. Rosa randomly picks two marbles from the bag. What is the probability that Rosa picks two white marbles?

Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

17. Sara is ordering a deli platter for a party. The deli offers 9 kinds of meats. Sara must select 4 kinds for the platter. How many different platters can Sara choose?

18. Armando has 12 antique books. He will select 3 books and put one up for auction each day for three days. How many different ways can he do this?